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A Detailed Study on Deflection of Fixed Beams Connected to Three Different Types of Building Structures due to the Pressure Applied on the Beams

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Abstract- Nowadays there is a huge trend on using timber roof structure with traditional joineries in hotels constructions because of its advantages like flexibility, durability, beautiful appearance, strength, eco-friendly, energy efficient, and aesthetical freedom. The king post and beam joinery is one of traditional timber joinery that commonly used when designing cabanas, summer houses, greenhouses and lobby in hotels. There is a standard measurement system is used by carpenters when design a roof structure and one of the basic measurements is the distance between two king-post trusses. Generally, carpenters in Sri Lanka use 8-10 feet as the range for the distance between two king-post trusses.

In this study, an attempt will be made to investigate the deflection of a fixed ridge beam due to pressure applied on the beam and from that to detect the optimum distance between two king post trusses within the general range and also find the optimum beam size that can be used to design the king post beam joinery more effective way. The study is intended to do analysis on three different types of building structures; Cabana, Lobby, and Summer house. In this study, a new model is generated to calculate the deflection of a fixed beam by using the Euler-Bernoulli Differential equation and Fourier series. The study is used only clay tiles as the roof covering materials and Kempas wood as the material for beams, king posts and rafters.

According to results for the cabana, the optimum distance between two king post trusses among the general range is 9 feet and the optimum beam size is 2x6 inch^2. The optimum distance between two king-post trusses among the general range is 8.5 feet and the optimum beam size is 3x6 inch^2 for the lobby. When considering the results for the summer house, the optimum distance between two king-post trusses among the considered range is 6.5 feet and the optimum beam size is 2x6 inch^2.

Index Terms- King post, Fixed ridge beam, Euler-Bernoulli theory, Fourier series

I. INTRODUCTION

The beams are structural elements that resist loads applied laterally to their axis and they typically transfer loads imposed along their length to their endpoints where the loads are transferred to walls, columns, foundations, and so on. A joint is an area where two separate pieces connect. In timber framing, there are many different types of joints and connections. Traditional joinery is the classic way to connect timbers in post & beam and timber frame structures. The king post and beam joinery are one of traditional joinery and it is an elegant and beautiful style of construction.

For this study, the simplest structure of king-post truss used and king-post and beam joinery connected by using mortise and tenon. Since both free ends are restrained against rotation and vertical movement, the ridge beam is a fixed.

II. RESEARCH ELABORATION

Euler-Bernoulli Differential Equation

Euler-Bernoulli Beam theory was developed around 1750 by Leonhard Euler and Daniel Bernoulli. It is a simple and effective method to calculate the behavior of beams when a load is applied.

$$\frac{d^4 y(x)}{dx^4} = \frac{1}{EI} q(x)$$

where y(x) - Deflection function. (m)

q(x) - Deflection pressure per unit length at point x(N)

E - Modulus of Elasticity of the beam. (Nm^{-2})

I - Moment of inertia of the beam. (m^4)

Modulus of Elasticity of wood

The modulus of elasticity is defined as the following equation.

$$E = \frac{\sigma}{\varepsilon}$$

where ε = Strain (no unit or %)

σ = Stress (MPa)

Its SI unit is the Pascal (Pa) or N/m^2 and the practical units used are Mega-Pascal (MPa or N/mm^2) or Giga Pascal (GPa or kN/mm^2).

Moment of inertia of the beam

For this study, the moment of inertia for a rectangular shape body is used.

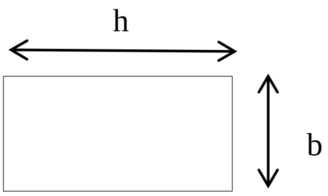

Figure 1: Rectangular shape body

Moment of inertia for rectangular shaped body,

$$I = \frac{bh^3}{12}$$

Deflection of a fixed beam under a lateral load

Consider a fixed beam is shown in the following figure 4.1. It is uniformly loaded by a load q per unit length. The deflection $y(x)$ of the beam is sought. The axis of the beam deflects from its initial position under the action of applied forces.

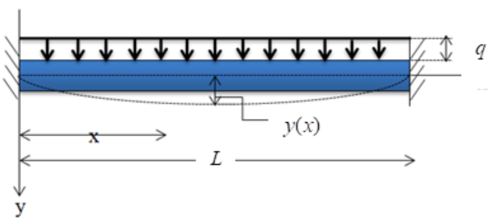

Figure 2: A fixed beam under a lateral load

The function $y(x)$ is satisfied with the differential equation

$$\frac{d^4 y(x)}{dx^4} = \frac{1}{EI} q(x) \longrightarrow (1)$$

where 1/*EI* is the rigidity of the beam.

The general expression for $y(x)$ which makes y and $\frac{dy}{dx}$ zero at both ends is

$$y(x) = \sum_{n=1}^{\infty} a_n \left\{ -\left(\frac{x^3}{L^3} - \frac{2x^2}{L^2} + \frac{x}{L} \right) - (-1)^n \left(\frac{x^3}{L^3} - \frac{x^2}{L^2} \right) + \frac{1}{n\pi} \sin\left(\frac{n\pi x}{L} \right) \right\}$$

Assuming the validity of the fourfold by term-by-term differentiation, obtain

$$\frac{d^4 y}{dx^4} = \sum_{n=1}^{\infty} a_n \left(\frac{n^3 \pi^3}{L^4} \right) \sin\left(\frac{n\pi x}{L} \right) \longrightarrow (2)$$

Then expand $q(x) = q$ into the Fourier sine series

$$q(x) = q = \sum_{n=1}^{\infty} q_n \sin\left(\frac{n\pi x}{L} \right) \longrightarrow (3)$$

where $q_n = \frac{2}{L} \int_0^L q \sin\left(\frac{n\pi x}{L} \right) dx$ n=1, 2, 3…

$$q_n = \frac{2}{L} \int_0^L q \sin\left(\frac{n\pi x}{L} \right) dx = \frac{-2q}{L} \left[\frac{\cos\left(\frac{n\pi x}{L} \right)}{\left(\frac{n\pi}{L} \right)} \right]_0^L = \frac{2q}{n\pi} [1 - \cos(n\pi)]$$

$$q_n = \begin{cases} \frac{4q}{n\pi} & \text{when n is odd} \\ 0 & \text{when n is even} \end{cases}$$

By (1), (2) and (3)

$$\sum_{n=1}^{\infty} a_n \left(\frac{n^3 \pi^3}{L^4} \right) \sin\left(\frac{n\pi x}{L} \right) = \frac{1}{EI} \sum_{n=1}^{\infty} q_n \sin\left(\frac{n\pi x}{L} \right)$$

When n is even, $a_n = 0$ and

when n is odd, $a_n \left(\frac{n^3 \pi^3}{L^4} \right) = \frac{1}{EI} \frac{4q}{n\pi}$

$$a_n = \frac{4qL^4}{EIn^4\pi^4}$$

Hence

$$y(x) = \sum_{n=1}^{\infty} a_n \left\{ -\left(\frac{x^3}{L^3} - \frac{2x^2}{L^2} + \frac{x}{L} \right) - (-1)^n \left(\frac{x^3}{L^3} - \frac{x^2}{L^2} \right) + \frac{1}{n\pi} \sin\left(\frac{n\pi x}{L} \right) \right\}$$

$$y(x) = \frac{4qL^4}{EI\pi^4} \sum_{\substack{n=2i+1 \\ i\in N}}^{\infty} \frac{1}{n^4} \left(\frac{x^2}{L^2} - \frac{x}{L} + \frac{1}{n\pi} \sin\left(\frac{n\pi x}{L} \right) \right) \longrightarrow (A)$$

Maximum Deflection Area Coefficient (MDAC)

In this study, a new coefficient named as Maximum Deflection Area Coefficient is generated to find the optimum beam sizes that suitable to build cabana, lobby and summer house. It is the multiplication of the cross-sectional surface area of the beam and maximum deflection of the beam.

A= Cross-sectional surface area

D= Maximum deflection of the beam

Maximum Deflection Area Coefficient = $A \times D$

Here optimum size beam introduced as the beam which has the lowest value for Maximum Deflection Area Coefficient.

Following reactions are considered for all building structures.

Reaction on the beam by rafter

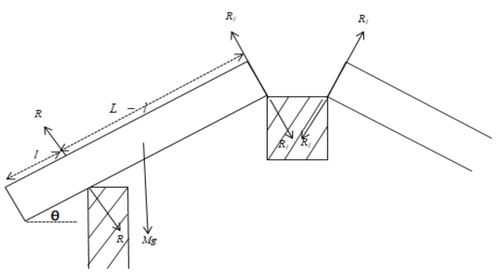

Figure 3: A roof structure

Following symbols are represented different reactions types of the roof structures.

$R -$ The reaction on wall beam by a rafter, $R_1 -$ The reaction on beam by rafter, $l -$ Length of the eave, $N -$ Number of rafters in one side of the beam, $q_1 -$ The reaction on the beam per unit length by rafter, q_2 –The weight of the beam per unit length, $q_3 -$ The reaction on the beam per unit length by support.

The moment around the wall beam

$$R_1 \times (L - l) = M \times \cos\theta \times \left(\frac{L}{2} - l \right)$$

$$R_1 = \frac{M \times \cos\theta \times \left(\frac{L}{2} - l \right)}{L - l}$$

The downward reaction on the beam by rafters=

$$R_3 = 2 \times R_1 \times \cos\theta = 2 \times (\cos\theta)^2 \times \frac{M \times \left(\frac{L}{2} - l \right)}{L - l}$$

The reaction on the beam per unit length by rafter, $q_1 =$

$$\frac{2 \times (\cos\theta)^2 \times \frac{M \times \left(\frac{L}{2} - l \right)}{L - l}}{\left(\frac{2}{39.37} \right)}$$

The weight of the beam per unit length = $q_2 = \frac{W}{D}$

Reaction on the beam by king post

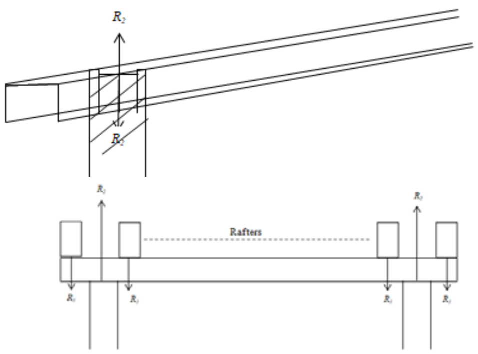

Figure 4: The king post and the beam joinery

The total reaction on the beam by both rafters and king posts= $R_2 = \left(\frac{N \times R_3 + W}{2} \right)$

The reaction on the beam per unit length by support =

$$q_3 = \frac{R_2}{3} \times 39.37$$

Following measurements are used for each building structures to obtain final results.

Table I: Measurements of all building structures

The building type	The length of the beam(m)	The number of rafters	The number of king post trusses	The angle between beam and rafter
Cabana	5.03	11	2	39.81^0
Lobby	10.01	21	4	39.81^0
Summer house	2.54	6	2	36.87^0

The selected region for the distance between two king post trusses of the cabana and the lobby is 7.5-10.5 feet and since the summer house is a small scale size of the building then the selected region for the distance between two king post trusses is 4.5-7 feet.

III. RESULTS

The MATLAB software is used to obtain the following results. In this study, the optimum distance between two king post trusses among the general range is obtained directly by using the minimum deflection value for each beam size. The optimum beam size is obtained by using a new coefficient named as Maximum Deflection Area Coefficient. The beam which has a minimum coefficient value is taken as the optimum beam size. All results are obtained by using Kempas wood for roof structures.

Table II, III, and IV are depicted the maximum deflection of the beam for all type of building that has different beam sizes and different distance between two king-post trusses. Table V is showed the Maximum Deflection Area Coefficient for all building structures.

Table II: The maximum deflection of the beam for the cabana

Type of building	The Cross-sectional surface area of the beam ($inch^2$)	The distance between two king post trusses (feet)	The maximum deflection of the beam (meter)
Cabana	2x4	7.5	0.0056
		8	0.0039
		8.5	0.0022
		9	0.000303
		9.5	0.0017
		10	0.0037
		10.5	0.0059
	2x5	7.5	0.0029
		8	0.002
		8.5	0.0011
		9	0.0001619
		9.5	0.0008586
		10	0.0019
		10.5	0.0031
	2x6	7.5	0.0017
		8	0.0012
		8.5	0.0006645
		9	0.00009824
		9.5	0.0004995
		10	0.0011
		10.5	0.0018
	3x4	7.5	0.0038
		8	0.0027
		8.5	0.0015
		9	0.0002213
		9.5	0.0011
		10	0.0025
		10.5	0.004
	3x5	7.5	0.002
		8	0.0014
		8.5	0.0007872
		9	0.000121
		9.5	0.0005823
		10	0.0013
		10.5	0.0021
	3x6	7.5	0.0012
		8	0.0008359
		8.5	0.0004661
		9	0.0000738
		9.5	0.0003403
		10	0.0007737
		10.5	0.0012

Table III: The maximum deflection of the beam for the lobby

Type of building	The Cross-sectional surface area of the beam (inch^2)	The distance between two king post trusses (feet)	The maximum deflection of the beam (meter)
Lobby	2x4	7.5	0.0985
		8	0.0545
		8.5	0.0078
		9	0.0417
		9.5	0.0939
		10	0.1489
		10.5	0.2066
	2x5	7.5	0.0508
		8	0.0281
		8.5	0.0039
		9	0.0217
		9.5	0.0488
		10	0.0772
		10.5	0.1071
	2x6	7.5	0.0564
		8	0.0310
		8.5	0.0041
		9	0.0244
		9.5	0.0545
		10	0.0862
		10.5	0.1190
	3x4	7.5	0.0666
		8	0.0367
		8.5	0.0049
		9	0.0288
		9.5	0.0644
		10	0.1018
		10.5	0.1410
	3x5	7.5	0.0345
		8	0.0189
		8.5	0.0023
		9	0.0152
		9.5	0.0337
		10	0.0531
		10.5	0.0735
	3x6	7.5	0.0201
		8	0.0110
		8.5	0.0013
		9	0.0090
		9.5	0.0198
		10	0.0312
		10.5	0.0445

Table IV: The maximum deflection of the beam for the summer house

Type of building	The Cross-sectional surface area of the beam (inch^2)	The distance between two king post trusses (feet)	The maximum deflection of the beam (meter)
Summer house	2x4	4.5	0.0047
		5	0.0036
		5.5	0.0023
		6	0.0009506
		6.5	0.0005239
		7	0.0021
	2x5	4.5	0.0025
		5	0.0019
		5.5	0.0012
		6	0.0004877
		6.5	0.0002766
		7	0.0011
	2x6	4.5	0.0014
		5	0.0011
		5.5	0.0006975
		6	0.0002824
		6.5	0.0001646
		7	0.0006435
	3x4	4.5	0.0032
		5	0.0024
		5.5	0.0016
		6	0.0006361
		6.5	0.0003706
		7	0.0014
	3x5	4.5	0.0017
		5	0.0013
		5.5	0.0008142
		6	0.0003268
		6.5	0.0001981
		7	0.0007605
	3x6	4.5	0.0008807
		5	0.0006246
		5.5	0.003554
		6	0.0002755
		6.5	0.0002127
		7	0.0005065

Table V: Maximum Deflection area Coefficient for all building structures

Type of the building structure	The cross-sectional surface area of the beam ($inch^2$)	The maximum deflection of the beam (inch)	Maximum Deflection Area Coefficient ($inch^3$)
Cabana	2x4	0.0012	0.0096
	2x5	0.0064	0.0640
	2x6	0.0039	0.0468
	3x4	0.0087	0.1044
	3x5	0.0048	0.0720
	3x6	0.0029	0.0522
Lobby	2x4	0.3079	2.4632
	2x5	0.1535	1.5350
	2x6	0.1614	1.9368
	3x4	0.1929	2.3148
	3x5	0.0906	1.3590
	3x6	0.0512	0.9216
Summer house	2x4	0.0109	0.0872
	2x5	0.0058	0.0580
	2x6	0.0034	0.0408
	3x4	0.0077	0.0924
	3x5	0.0041	0.0615
	3x6	0.0025	0.0450

IV. CONCLUSIONS

According to results, the optimum distance between two king post trusses among the general range is 9 feet, 8.5 feet and 6.5 feet for cabana, lobby and summer house respectively. According to Table V, the optimum size of the beam for cabana is 2x6 $inch^2$, for lobby is 3x6 $inch^2$ and for summer house is 2x6 $inch^2$.

All the results are obtained by considering the deflection values of the beam under some limitations from the practical situation. Following limitations are used for the study.: The beam is homogeneous material that has the same modulus of elasticity in tension and compression, The cross-sectional surface area of the beam is uniform, The beams are rectangular timber beams and it has a longitudinal plane of symmetry, Any section of a beam that is a flat plane before the beam deforms will remain a flat plane after the beam deforms, Any section of a beam that is perpendicular to the neutral axis before the beam deforms will remain perpendicular to the neutral axis after the beam deforms, The roof covering material here used only clay tiles, In this study based on fixed supported beam structure that has only beams king posts and rafters, Only consider the weights of beams, rafters and clay tiles.

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