Developing a New Paper Structure For The G.C.E.(O/L) Mathematics Paper Using Graph Theory

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Abstract- This study focuses on developing a new paper structure for the G.C.E.(O/L) Mathematics paper using Graph Theoretical concepts. The graph theoretical model is built based on the competency levels of the O/L (Ordinary Level) Mathematics syllabi and their weights. Centrality measures such as Weighted Degree Centrality, Betweenness Centrality, Authority and Hub are used to analyze this model. The open source software Gephi is used to do the analysis. The new paper structure is designed based on the dominancy of each competency and the suggested weight of each Mathematical Theme.

Index Terms- G.C.E.(O/L) Mathematics, Graph theory, Centrality measures

1. INTRODUCTION

1.1 Background of Mathematics Education in Sri Lanka

The modern education system in Sri Lanka was introduced by the British in the 19th century. Since then, Mathematics has been taught in schools in different levels and in different modules by either making it compulsory or optional. But in 1972, Mathematics was made compulsory up to SSC (Secondary School Certificate) where nowadays call as G.C.E.(O/L). After several amendments in the curricula, in 2006, the Ministry of Education (MOE) in Sri Lanka introduced the Competency Based Curriculum incorporating the 5E learning cycle (5E- Engage, Explore, Explain, Elaborate and Evaluation). Parallel to the curriculum reformation, questions relating to real life scenarios were added to the O/L paper from 2008. Mathematics taught in junior secondary (year 6 to 9) has many applications in day to day activities. But most of the lessons covered in year 10 and 11 lay out a foundation for higher mathematics.

1.2 Current Stage of G.C.E.(O/L) Mathematics

According to statistics released by Department of Examinations (DOE) the failure rate from 2011 to 2015 was in between 40% to 45%. The pass rate has improved slightly after 2015, but still the failure rate is in between 30% to 40%. In 2017, 53.68% of students have scored less than 40 marks in the O/L examination. High failure rate in the G.C.E.(O/L) Mathematics is a huge problem that the educators and the authorities should look into. MOE has taken a number of steps to reduce the failure rate in O/L. Back in 1999, DOE has had two examination papers at two levels of difficulty. Then in 2001, Multiple Book Option (MBO) was introduced by MOE. Again in 2005, DOE conducted a mock examination for Mathematics. But all these initiatives did not last long. According to Mampitiya (2014) the O/L results can be improved by a properly designed examination paper.

2. METHODOLOGY

The current mathematics syllabus is built on 31 competencies. When generating the graph theoretical model, these competencies were taken as nodes and the links between each competency were represented by edges. These edges were given weights based on how much they are connected with each other. The dominancy of each competency in the directed weighted graph was analyzed using Gephi under the statistics; weighted degree, weighted in-degree, weighted out-degree, betweeness centrality, authority and hub. Points were given for each competency (node) according to their value of the statistic. Most dominant competencies were identified and percentage (suggested weight) of each mathematical theme was calculated based on Node Values. A new paper structure was designed based on the suggested weights.

3. RESULTS AND DISCUSSION

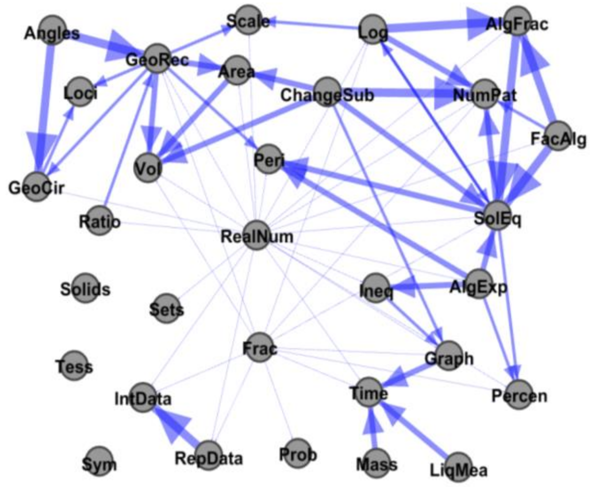

Figure 3.1: Graph Theoretical Model

Figure 3.1 is the graph obtained after considering competencies as nodes and the links between them as edges. It can be clearly seen that *Solids*, *Tessellation* and *Symmetry* have no link with the other nodes. These three topics are taught in year 6 to 8 and the content is not strong to be tested in the O/L exam. Therefore the above three nodes were removed from the further analysis.

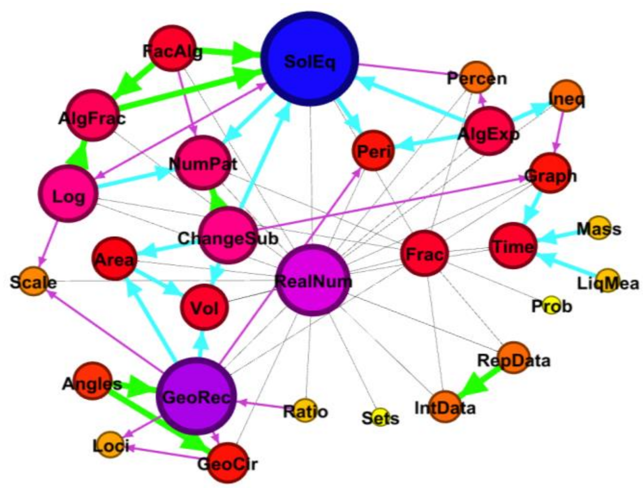

Figure 3.2: Ranking using Weighted Degree Centrality

According to the weighted degree centrality *Solving Equations* is the dominant node. *Geometry-Rectilinear* and *Real Numbers* are the next two important nodes. But on the other hand, *Sets* and *Probability* have only one link with the others. A node with high weighted degree indicates that the node is having a lot of links or strong links with the other nodes. Therefore, topics like *Solving Equations, Geometry-Rectilinear* and *Real Numbers* should be given more attention in teaching and learning process.

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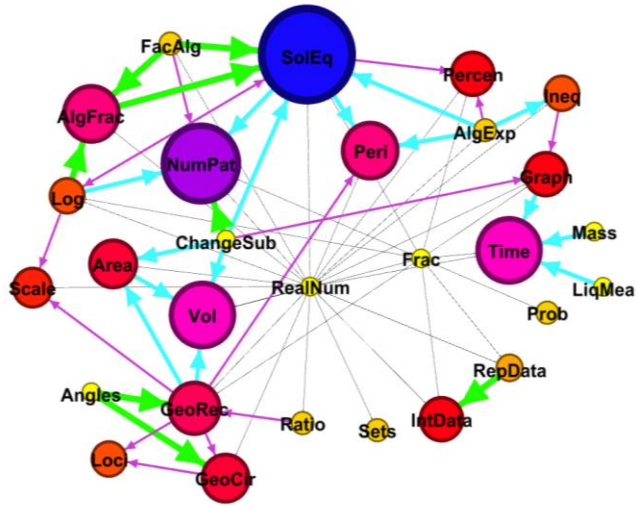

Figure 3.3: Ranking using Weighted In-degree Centrality

When the weighted in-degree is considered, *Solving Equations* has the highest in-degree centrality. *Number Patterns, Volume* and *Time* are some of the next few important nodes. Since no topics are applied in *Real Numbers, Fractions, Changing the Subject, Angles, Mass* and *Liquid Measures*, they have no weighted in-degree. The questions that are set on these six topics can be addressed using what is taught under that topic, because they have no incoming links. Questions on topics that are having high weighted in-degree might be difficult for the students as they need to be familiar with all the topics linked into those topics.

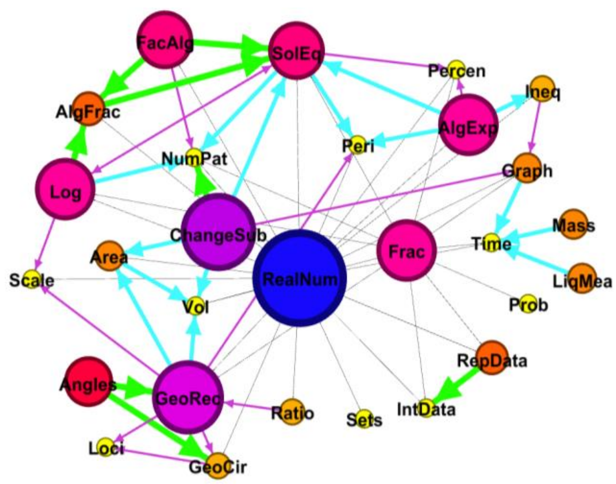

Figure 3.4: Ranking using Weighted Out-degree Centrality

The node *Real Numbers* is dominant when the weighted out-degree is considered. The next two important nodes are *Changing the Subject* and *Geometry_ Rectilinear*. It is possible to form individual questions on topics such as *Number Patterns*, *Volume*, *Time, Perimeter*, *Percentages, Interpretation of Data, Scale Diagrams* and *Loci Construction* because the weighted out-degrees of those nodes are zero. Studying thoroughly on topics having high weighted out-degree will be very much beneficial for the students as those are applied in many other topics linked.

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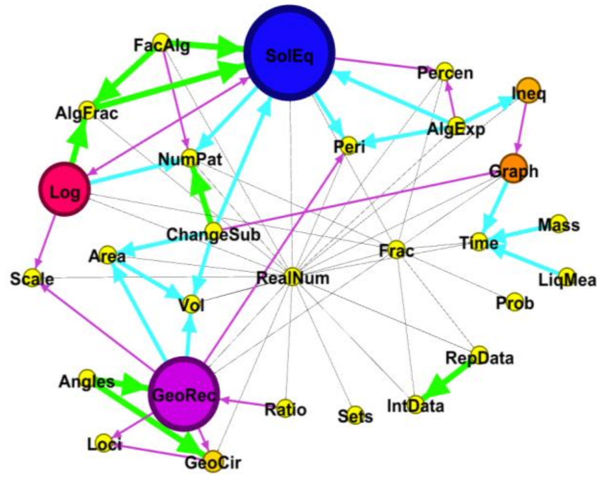

Figure 3.5: Ranking using Betweenness Centrality

There are few nodes acting as bridges in this model. Among those *Solving Equations* is the most dominant node. Nodes acting as bridges are important to merge topics when generating questions. The betweenness centrality of *Solving Equations* is very high compared to other nodes. Therefore, students should be familiar with the lessons that are covered around *Solving Equations.* When the three most dominant nodes are considered, it is noticeable that they belong to three different themes in the syllabus. *Solving Equations* belongs to Algebra, *Geometry_Rectilinear* belongs to Geometry and *Logarithm* belongs to Numbers.

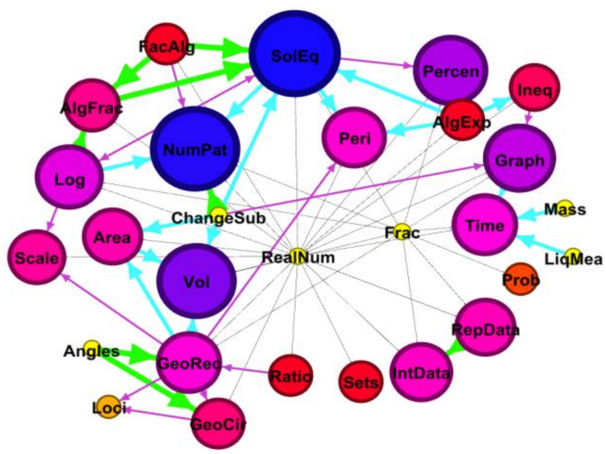

Figure 3.6: Ranking using Authority values

Nodes that contain useful information on a topic of interest are defined as authorities. But in this setting, a node having high authority value emphasize that, that particular topic uses the information of many more topics. *Solving Equations*, *Number Patterns* and *Volume* are the nodes that are having high authority values correspondingly. When the students study, they should practice more on topics having high authority values. The paper setters can select these high authoritative topics to test not only on the selected topic but the topics linked into those as well, rather than giving another question to test the linked topics. This can be used to reduce the number of questions in a test paper.

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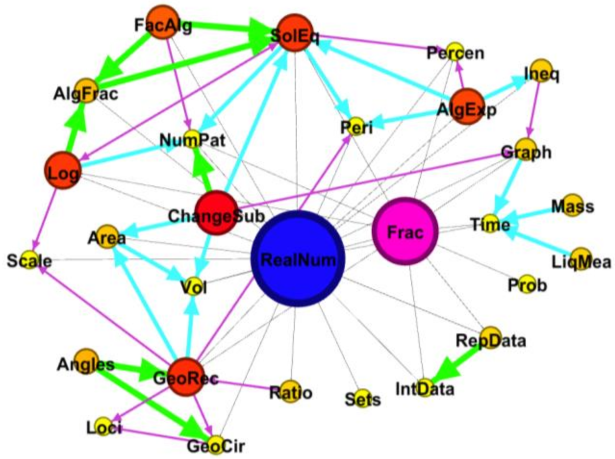

Figure 3.7: Ranking using Hub values

Hubs are nodes that tell where the best authorities can be found. The hub value of a node is high, if it is connected to a good authority. According to figure 3.7, *Real Numbers* and *Fractions* are the two main hubs. Their hub values are high, not only because they have lot of links but also because they are linked into good authorities such as *Solving Equations, Number Patterns* and *Volume.*

After analyzing the graph based on the six statistics that were selected, nodes were given points according to their value of the statistic. After total points of each node was calculated, *Solving Equations* was identified as the most dominant node. *Geometry_ Rectilinear*, *Logarithms*, *Number Patterns* and *Real Numbers* lie next in the sequence. Students, teachers and paper setters can identify the above topics as most powerful areas in the syllabus.

Questions in the O/L mathematics paper fall under six themes namely, *Numbers*, *Measurements*, *Algebra*, *Geometry*, *Statistics* and *Sets & probability*. Therefore the percentages of the six mathematical themes were calculated based on the total node values.

Table 3.1: Percentages of the mathematics themes

Theme	Node value Total	Percentage
Numbers	226	25.22321
Measurements	188	20.98214
Algebra	287	32.03125
Geometry	125	13.95089
Statistics	51	5.691964
Sets & Probability	19	2.120536

Since the percentages of the themes *Statistics* and *Sets & probability* are both less than 10%, the two themes were combined and called as *Statistics.*

Table 3.2: Comparison between the existing figures and the suggested figures

Theme	Percentage in the existing structure	Suggested Percentage
Numbers	22%	25%
Measurements	17%	21%
Algebra	19%	32%
Geometry	21%	14%
Statistics	21%	8%

According to table 3.2, the weight of *Algebra* should be increased up to 32% and it is a major change that should be done while making the exam paper. Also the weight of *Statistics* should not be more than 8% and weight of *Geometry* should be decreased by 7%.

Table 3.3: Existing Structure of the Mathematics paper

<table>
<tr><th colspan="4">32 - Mathematics</th></tr>
<tr><th colspan="2">Paper I
2 hours
100 marks</th><th colspan="2">Paper II
3 hours
100 marks</th></tr>
<tr><th>Part A</th><th>Part B</th><th>Part A</th><th>Part B</th></tr>
<tr><td>25 short answer questions</td><td>5 Structured questions</td><td>6 Structured questions
Should select 5 to answer</td><td>6 Structured questions
Should select 5 to answer</td></tr>
<tr><td>(25×2marks)</td><td>(5×10marks)</td><td>(5×10marks)</td><td>(5×10marks)</td></tr>
<tr><td>Questions from the entire syllabus</td><td>No questions on Algebra & Geometry</td><td>No questions on Geometry
3 Questions on Algebra
3 Questions out of Numbers, Measurements, Statistics and Probability</td><td>No questions on Algebra
3 Questions on Geometry
3 Questions out of Numbers, Measurements, Statistics and Probability</td></tr>
</table>

The table 3.3 gives an overview of the existing structure of the mathematics paper.

Table 3.4: Compulsory subjects with their time duration

Compulsory Subject	Time Duration for both paper 1 & paper 2
Religion	3 hours
First Language *	3 hours
English	3 hours
Mathematics	5 hours
History	4 hours
Science	4 hours

*First language consists of a third paper which is a literature paper in which the duration is 2 hours.

According to the above table it is clear that students have to be occupied more on mathematics. This again shows that mathematics is given more weight amongst the other subjects. So at least time duration for mathematics should be reduced up to 4 hours for the weights of the subjects to be fairly distributed.
The proposed structure for the O/L mathematics paper is as follows.

Table 3.5: Suggested Paper Structure

Paper 1	Paper 2
100 marks	100 marks
2 hours	2 hours
50%	50%
Answer all questions	Answer all questions
Type of questions: • Short answer • Structured	Type of questions: • Structured • Un-Structured
Level of Complexity of the questions: • Low • Moderate	Level of Complexity of the questions: • Moderate • High
Distribution of Themes : • Numbers – 25 marks • Measurements – 21 marks • Algebra – 32 marks • Geometry – 14 marks • Statistics – 8 marks	Distribution of Themes : • Numbers – 25 marks • Measurements – 21 marks • Algebra – 32 marks • Geometry – 14 marks • Statistics – 8 marks

Both Short answer questions (SAQ) and Structured questions (SQ) should be included in the first paper to maintain the standard of the paper, and in the second paper SQs and Unstructured questions (USQ) can be included, where USQs can be used to filter the best students to do Advanced Level Mathematics. Also the paper setters should be given the freedom to decide the number of questions that can be put in the exam paper while maintaining the weights proposed in this article and maintaining the standard of the paper.

4. CONCLUSION

According to the centrality analysis *Solving Equations* is the most dominant competency. *Geometry_Rectilinear* and *Logarithms* are next in order. Students can use the graph theoretical model used for the analysis to see how the topics are connected, how important each topic is etc. and prepare themselves better for the exam. According to the analysis, more weight should be put on the theme *Algebra* and one fourth of the paper should be based on the theme *Numbers.* It was also suggested to combine the themes *Statistics* and *Sets & Probability*. The time duration of the mathematics paper was suggested to be reduced up to 4 hours.
In the proposed first paper, there should be short answer questions and structured questions with low and moderate complexity and structured and unstructured questions with moderate and high complexity in the second paper. But both the papers should carry questions on *Numbers* with 25 marks, *Measurements* with 21 marks, *Algebra* with 32 marks, *Geometry* with 14 marks and *Statistics* with 8 marks. It should also be compulsory to answer all the questions.

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