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A Study on Graph Theory Properties of Constellations

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Abstract. A constellation is a group of stars that are considered to form imaginary outlines or meaningful patterns on the celestial sphere. The 88 constellations are formally defined regions of the sky together covering the entire celestial sphere. In this study, all the constellations have been analysed using many graph theory properties such as chromatic number, clique number, Hamiltonian and Eularian nature and network properties such as hub value, clustering coefficient and centrality measures etc. Further, the constellation patterns were classified in few ways considering their graph properties, such as bipartiteness, cyclic acyclic nature, regularity etc. This analysis helps to look at star constellations in graph theoretic way and understand the importance of each star within the constellations graph.
Keywords: Graph Theory, Stars, Constellations, Connectedness

Introduction

The subject of this research consists of identification of the graph theoretical properties of 88 International Astronomical Union (IAU) star constellations. A constellation is formally defined as a region of the celestial sphere, with boundaries defined by the IAU. For many years, handful people have seen constellations in mathematical ways, especially in graph theoretical aspects. Graph theory is a mathematical field which can be used to observe the properties of bonds in nature. A previous elementary graph-theoretical analysis on star patterns was carried out by R. S. Rao (2014) and provided some elementary properties of graph theory to study the similarities and dissimilarities of Big Dipper (an asterism) and Euler graph. H. Ueda and M. Itoh (1997) have proposed a graph theoretical approach for quantifying the galaxy distribution in the universe using constellations. Also, M. B. Melou and G. Schaeffer (2000) have explained constellations and their relations by defining and enumerating the family of trees and describing the correspondence between these trees and constellations. However, in this paper we will discuss the basic graph properties such as order, size, chromatic number, connectivity, and clique number etc. Also, properties describe the nature of a graph such as cyclic/acyclic, bipartite/non-bipartite, completeness, Eulerian/Hamiltonian and regularity are observed and then the constellations will be classified according to their properties. Also, importance of each star within the constellation is identified using few network properties such as degree, eccentricity, clustering coefficient, centrality measures, etc.

Summary of Graph Theory Properties in a Constellation Graph

In this section the graph theory concepts which are applied in this study are summarized.

Definition 1

Constellation Graphs G=(S, C):

A constellation graph is a graph, constructed by connecting nearest-neighbour pairs of stars. Stars(S), lines that connect between nearest neighbours(C), and constellations (G), correspond to vertices, edges, and graphs respectively.

A walk is termed closed if $s_1 = s_p$, where s_1 and $s_p$are first and last vertices of the walk and open otherwise. A walk is called a trail if all of its connections are distinct, and a path if all of its stars (and necessarily all of its connections) are distinct. A closed walk with at least three stars, and all its stars distinct, (except for the first and the last) is called a cyclic constellation graph. A constellation graph $G = (S, C)$ is said to be connected if every pair of stars in G is connected by a path. A constellation graphwithout cycles is called acyclic. A connected graph $G = (S, C)$, which contains a sub constellation graph which is the tree $T = (S', C')$. A tree is a connected acyclic graph. If $S = S'$, then, T is said to be a spanning constellation tree of graph$G$. The numbers of stars and connections are called the order and the size of graph G. The number of connections incident with a star s in a graph is called the degree (Deg) of s, and is written as $d(s)$. A complete constellation graph is a simple undirected graph in which every pair of distinct stars is connected by a unique line. Also, the clique number of a constellation graph is the number of stars in a maximum clique of the graph. The length of the shortest cycle in the constellation is called a Girth. Chromatic number is the smallest number of colors for the stars in a proper coloring and Chromatic index is the smallest number of colors for the connections in a proper coloring. A subset D of S such that every star not in D is adjacent to at least one member of $D$is called the dominating set and the number of stars in a smallest dominating set for G is defined as the dominating number. An independent set is a set of stars in a constellation such that no two of them are adjacent. The largest size of an independent set of stars is termed as the independent number. A Eulerian constellation graph is a circuit which visits every connection exactly once. A Hamiltonian constellation graph is a cycle in a constellation that visits each star of the constellation exactly once. Also, a regular constellation graph is a graph where each star has the same number of neighbors.

For the identification of important nodes (stars) within a constellation, following properties are also considered. In a graph, the longest of the shortest path lengths between pairs of stars is called as the diameter. Also, the radius is the shortest of the shortest path lengths between pairs of stars. The eccentricity (Ecc) of the star $s$is the maximum distance from s to any vertex. Clustering coefficient is a measure of the degree to which stars in a graph tend to cluster together. In real networks, this measures the "all-my-friends-know-each-other" property. This is sometimes described as the friends of my friends are my friends. The clustering coefficient of the i^{th}star is:

$$C_i = \frac{2e_i}{K_i(K_i - 1)}, \quad (1)$$

where K_i is the number of neighbours of the i^{th}star, and e_i is the number of connections between these neighbouring stars. The Closeness Centrality (CC) is the average length of the shortest path between the stars and all other stars in the constellation graph. Thus, the more central a star is, closer to all other stars.

$$C(x) = \frac{1}{\sum_y d(y,x)}, \quad (2)$$

where $d(y, x)$ is the distance between stars x and y. Harmonic Centrality (HC) of a (not necessarily connected) constellation graph, reverses the sum and reciprocal operations in the definition of closeness centrality:

$$H(x) = \sum_{y \neq x} \frac{1}{d(y,x)}, \quad (3)$$

where $\frac{1}{d(y,x)} = 0$, if there is no path from y to x. Betweenness Centrality (BC) is the number of times a star acts as a bridge along the shortest path between two other stars. Stars that have a high probability to occur on a randomly chosen shortest path between two randomly chosen stars have a high betweenness.

$$C_{\boldsymbol{B}}(v) = \sum_{s \neq v \neq t \in S} \frac{\sigma_{st}(v)}{\sigma_{st}}, \quad (4)$$

where σ_{st} is total number of shortest paths from star s to star t and $\sigma_{st}(v)$ is the number of those paths that pass through v. Eigen Vector Centrality (EVC) is a measure of the influence of a star in a constellation graph. For a given constellation graph $G = (S, C)$ with $|S|$number of vertices let $A = a_{st}$ be the adjacency matrix, i.e. $a_{st} = 1$ if star$s$ is linked to star t, and $a_{st} = 0$ otherwise. The relative centrality score of stars can be defined as:

$$x_s = \frac{1}{\lambda}\sum_{t\in M(s)} x_t = \frac{1}{\lambda}\sum_{t\in G} a_{s,t}x_t, \quad (5)$$

where $M(s)$ is a set of the neighbouring stars of S and λ is a constant. With a small rearrangement this can be rewritten in vector notation as the eigenvector equation $Ax = \lambda x$. Graph Density (D) is the ratio of the number of connections C to the number of possible connections in a constellation graph with S nodes, given by the binomial coefficient $\binom{S}{2}$,

$$D = \frac{2(C-S+1)}{S(S-3)+2}, \quad (6)$$

The Authority Score (Aut) indicates the value of the star itself. How valuable information stored at that star is. The Hubs estimate the value of the connections outgoing from the star. It measures the quality of the connection between two neighbouring stars. Modularity (M) measures the strength of division of a constellation graph into modules (also called groups, clusters or communities). Constellation graphs with high modularity have dense connections between the stars within modules but sparse connections between stars in different modules.

$$M = \sum_{i=1}^{c} e_{ii} - a_i^2, \quad (7)$$

where e_{ij} is the fraction of connections with one end star in community i and the other in community j: $e_{ij} = \sum_{vw}\frac{A_{vw}}{2m}1_{v\in c_i}1_{w\in c_j}$ and a_i is the fraction of ends of connections that are attached to stars in community i: $a_i = \frac{k_i}{2m} = \sum_j e_{ij}$, where n and m are number of stars and connections respectively. For randomly selected v and $w$stars, $A_{vw} = 1$ if there exist a connection between v and w and $A_{vw} = 0$ otherwise. Average path length (APL) of a constellation graph is calculated by finding the shortest path between all pairs of stars, adding them up, and then dividing by the total number of pairs. This shows, on average, the number of steps it takes to get from one member of the network to another.

Methods

Identification of constellation graphs, analysis of the constellation graphs, theoretically and visually, and summarization of the results obtained by the analysis, are identified as the main activities in this study.

Identification of the Constellation Graphs

There are 88 constellations identified by the International Astronomical Union. As the first step of this study, patterns of all the 88 constellations were identified. The images for constellation patterns were taken from the website http://www.seasky.org/. Then the constellation patterns are turned into form of graphs by assigning numbers to every node.

Analysis of the Constellation Graphs

In order to apply a graph-theoretical approach to two-dimensional star constellations, graph theoretical properties of planar constellation graphs are identified. Here, basic graph properties are identified and the constellation graphs, corresponding complete graphs, line graphs and minimal spanning trees are generated by using the software, *Maple*. Few of network properties of constellation graphs are also identified using the software *Gephi*. The properties which are analysed by Maple, give an intuition about the whole graph and the properties identified by Gephi, give an intuition about the nodes of a graph.

Summarization of the Results

The obtained results are summarized according to selected graph properties. The basic graph properties and network properties which have been observed by Gephi and Maple are compared considering few properties by categorizing constellations graphs. The aim of this comparison is to identify the common properties of each category. The network properties are analysed and summarized in order to identify the most important nodes in the graph.

Results and Discussion

Summary of Properties of Connected Constellation Graphs

Thirty-two graphs out of 88 can be identified as connected and remaining planar constellation graphs are disconnected. Here, as the first step, basic graph and network properties of connected graphs are identified.

As an example, we consider the constellation *Antlia* (The Air Pump), and Figure 1 depicts the constellation map and generated graph of Antlia constellation. Also, Table 1 contains the several graph properties calculated for the nodes of Antlia graph. From the values from Table 1, we can see that *α Ant* is the most important star in the Antlia constellation. Similarly, the analysis conducted for each and every constellation graph and obtained the important star within every graph. Furthermore, Table 2 shows the graph properties for each connected constellation graph and contains abbreviated name for constellations, No. of Edges (E), No. of Vertices (N), Chromatic Number (CN), Chromatic Index (CI), Vertex Connectivity (VC), Edge Connectivity (EC), Clique Number (CN), Girth (G), Graph Rank (GR), Independence Number (IN)and graph nature (Acyclic (Ac), Tree (Tr), Completeness (Com), Eulerian (Eu), Hamiltonian (Hm) and Regularity (Rg)).

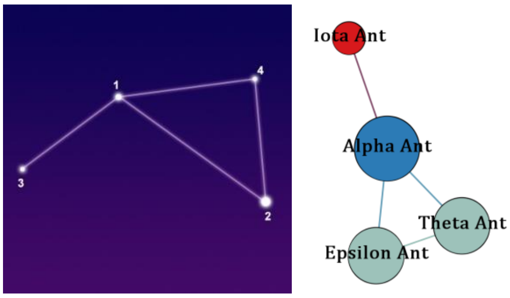

Figure 1. Antlia constellation map

Abbreviation	Ant
Genitive	Antliae
Symbolism	The Air Pump
Main stars	3
Bordering constellations	Hydra, Pyxis, Vela, Centaurus

Table 1. Graph Properties for nodes of Antlia

ID	Star	Deg.	Ecc.	CC	HC	BC	Aut.	Hub	M	Clu.	Tri.	EVC
1	*α Ant*	3	1	1	1	2	0.6116	0.6116	0	0.3333	1	1
2	*ε Ant*	2	2	0.75	0.8333	0	0.5227	0.5227	1	1	1	0.854
3	*ι Ant*	1	2	0.6	0.6667	0	0.2818	0.2819	0	0	0	0.4614
4	*θ Ant*	2	2	0.75	0.8333	0	0.5227	0.5227	1	1	1	0.854

Note. Deg: Degree, Ecc: Eccentricity, Aut: Authority Score, Hub: Hub Value, Clu: Clustering Coefficient, Tri: No. of Triangles

Table 2. Basic graph properties for connected constellations

Con.	E	N	CN	CI	VC	EC	C	G	GR	IN	Ac	Tr	Com	Eu	Hm	Rg
Ant	4	4	3	3	1	1	3	3	3	2	F	F	F	F	F	F
Apu	4	5	2	2	1	1	2	∞	4	3	T	T	F	F	F	F
Aql	11	10	3	4	1	1	3	3	9	5	F	F	F	F	F	F
Ara	8	8	3	3	1	1	2	7	7	4	F	F	F	F	F	F
Ari	8	9	2	4	1	1	2	∞	8	5	T	T	F	F	F	F
Cam	7	8	2	3	1	1	2	∞	7	4	T	T	F	F	F	F
Cea	3	4	2	2	1	1	2	∞	3	2	T	T	F	F	F	F
Cen	21	19	3	4	1	1	3	3	18	10	F	F	F	F	F	F
Cha	6	6	2	2	2	2	2	6	5	3	F	F	F	T	T	T
Cir	3	4	2	2	1	1	2	∞	3	2	T	T	F	F	F	F
CrA	5	6	2	2	1	1	2	∞	5	3	T	T	F	F	F	F
CrB	6	7	2	2	1	1	2	∞	6	4	T	T	F	F	F	F
Crv	5	5	2	3	1	1	2	4	4	3	F	F	F	F	F	F
Crt	8	8	2	3	1	1	2	4	7	4	F	F	F	F	F	F
Del	5	5	2	3	1	1	2	4	4	3	F	F	F	F	F	F
Equ	2	3	2	2	1	1	2	∞	2	2	T	T	F	F	F	F
For	4	4	2	2	2	2	2	4	3	2	F	F	F	T	T	T
Hor	6	7	2	2	1	1	2	∞	6	4	T	T	F	F	F	F
Ind	5	5	3	3	2	2	2	5	4	2	F	F	F	T	T	T
Lac	8	9	2	2	1	1	2	∞	8	5	T	T	F	F	F	F
Lmi	5	5	2	3	1	1	2	4	4	3	F	F	F	F	F	F
Lyn	7	8	2	2	1	1	2	∞	7	4	T	T	F	F	F	F
Men	3	4	2	2	1	1	2	∞	3	2	T	T	F	F	F	F
Mic	5	5	3	3	2	2	2	5	4	2	F	F	F	T	T	T
Mus	6	6	2	3	1	1	2	4	5	3	F	F	F	F	F	F
Pav	12	11	3	6	1	1	2	4	10	7	F	F	F	F	F	F
Scl	5	5	3	3	2	2	2	5	4	2	F	F	F	T	T	T
Sex	4	4	2	2	2	2	2	4	3	2	F	F	F	T	T	T
Tel	6	6	2	2	2	2	2	6	5	3	F	F	F	T	T	T
UMi	7	7	2	3	1	1	2	4	6	4	F	F	F	F	F	F
Vel	8	8	2	2	2	2	2	8	7	4	F	F	F	T	T	T
Vol	6	6	3	3	1	1	2	5	5	3	F	F	F	F	F	F

Here, in Table 3 contains the network properties for connected constellations such as Diameter (D), Radius (R), Average Degree (AD), Average Path Length (APL), Density (Den), Modularity (Mod), No. of Communities (C), Average Clustering Coefficient (ACC) and No. of Triangles (T).

Table 3. Network properties for connected constellations

Cons.	D	R	AD	APL	Den.	Mod.	C	ACC	T
Ant	2	1	2	1.333	0.667	0	2	0.778	1
Apu	4	2	1.6	2	0.4	0.219	2	0	0
Aql	5	3	2.2	2.489	0.244	0.269	3	0.104	1
Ara	4	3	2	2.178	0.286	0.281	3	0	0

Ari	4	2	1.778	2.389	0.222	0.367	3	0	0
Cam	5	3	1.75	2.571	0.25	0.306	4	0	0
Cea	3	2	1.5	1.667	0.5	0.167	2	0	0
Cen	9	5	2.1	4.005	0.111	0.554	4	0.056	1
Cha	3	3	2	1.8	0.4	0.167	3	0	0
Cir	3	2	1.5	1.667	0.5	0.167	2	0	0
CrA	5	3	1.667	2.333	0.333	0.26	3	0	0
CrB	6	3	1.714	2.667	0.286	0.319	3	0	0
Crv	3	2	2	1.6	0.5	0.08	2	0	0
Crt	5	3	2	2.357	0.286	0.281	3	0	0
Del	3	2	2	1.6	0.5	0.08	2	0	0
Equ	2	1	1.333	1.333	0.667	0	1	0	0
For	2	2	2	1.333	0.667	0	2	0	0
Hor	6	3	1.714	2.667	0.286	0.319	3	0	0
Ind	2	2	2	1.5	0.5	0.08	2	0	0
Lac	8	4	1.778	3.333	0.222	0.398	3	0	0
LMi	3	2	2	1.6	0.5	0.08	2	0	0
Lyn	7	4	1.75	3	0.2	0.378	3	0	0
Men	3	2	1.5	2.285	0.286	0.292	3	0	0
Mic	2	2	2	1.5	0.5	0.08	2	0	0
Mus	4	2	2	1.933	0.4	0.208	2	0	0
Pav	5	3	2.182	2.309	0.218	0.319	3	0	0
Scl	2	2	2	1.5	0.5	0.08	2	0	0
Sex	2	2	2	1.333	0.667	0	2	0	0
Tel	3	3	2	1.8	0.4	0.167	3	0	0
UMi	5	3	2	2.285	0.333	0.214	3	0	0
Vel	4	4	2	2.285	0.286	0.25	4	0	0
Vol	3	2	2	1.733	0.4	0.167	2	0	0

According to these properties in Table 3, Centaurs can be identified as the biggest connected constellation graph while the Equuleus constellation graph is the smallest. Antlia graph shows the highest density and highest clustering coefficient. Antlia, Aquila and Centaurs graphs tend to cluster since those contain complete sub graphs (triangles).

Summary of Properties of Disconnected Constellation Graphs

Fifty six of 88 constellation graphs can be identified as disconnected graphs and except Crux, every disconnected constellation graph has one connected component and isolated vertices. But only the connected component will be analysed since the graphs would not give more information with isolated vertices.

As an example, we consider the constellation *Andromeda* (The Princess of Ethiopia), and Figure 2 shows the constellation map and generated graph of Andromeda constellation. Also, Table 4 contains the several graph properties calculated for the nodes of Antlia graph. From the values from Table 4, we can see that δ *And* is the most important star in the Andromeda constellation.

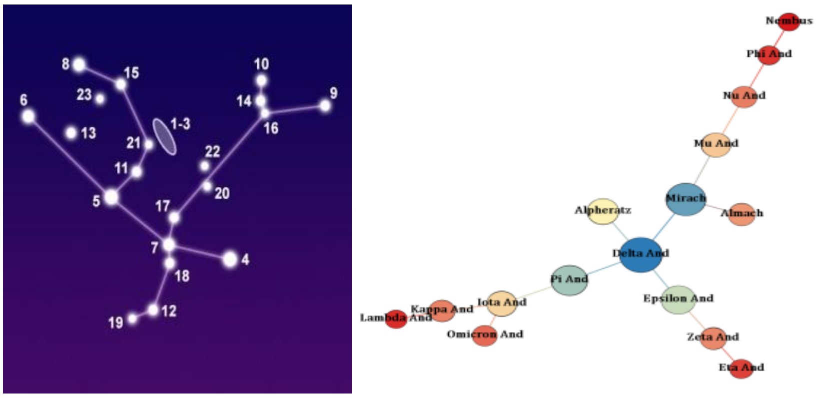

Figure 2. Andromeda constellation map

Abbreviation	And
Genitive	Andromedae
Symbolism	The Princess of Ethiopia
Main stars	16
Bordering constellations	Perseus, Cassiopeia, Lacerta, Pegasus, Pisces, Triangulum

Table 4. Graph properties for nodes of Andromeda

Id	Label	Deg	Ecc.	CC	HC	BC	Aut.	Hub	MC	Clu.	Tri.	EC
4	Alpheratz	1	6	0.2941	0.36	0	0.2202	0.2596	0	0	0	0.4162
5	Mirach	3	5	0.3659	0.48	54	0.3813	0.4494	0	0	0	0.7545
6	Almach	1	6	0.2727	0.3367	0	0.1927	0.1631	0	0	0	0.3175
7	**δ And**	4	5	0.4054	0.5356	77	0.6069	0.5149	0	0	0	1
8	Nembus	1	9	0.1765	0.2503	0	0.0263	0.031	3	0	0	0.1074
9	ο And	1	8	0.2308	0.3079	0	0.0982	0.1158	2	0	0	0.2342
10	λ And	1	9	0.1948	0.2692	0	0.0606	0.0514	2	0	0	0.1444
11	μ And	2	6	0.3061	0.4067	36	0.2515	0.2134	3	0	0	0.4651
12	ζ And	2	7	0.2542	0.3551	14	0.1751	0.1486	1	0	0	0.305
14	κ And	2	8	0.2381	0.3523	14	0.1202	0.1417	2	0	0	0.3063
15	φ And	2	8	0.2113	0.3213	14	0.0726	0.0616	3	0	0	0.2072
16	ι And	3	7	0.2941	0.4306	38	0.2707	0.2297	2	0	0	0.5256
17	π And	2	6	0.3488	0.4467	44	0.3185	0.3754	0	0	0	0.6503
18	ε And	2	6	0.3191	0.4211	26	0.2838	0.3345	1	0	0	0.5552
19	η And	1	8	0.2055	0.272	0	0.0635	0.0749	1	0	0	0.1391
21	ν And	2	7	0.2542	0.364	26	0.1176	0.1386	3	0	0	0.3112

The Table 5 contains the graph properties for each disconnected constellation graph and contains No. of edges, No. of vertices, Chromatic Number, Chromatic Index, Vertex Connectivity, Edge Connectivity, Clique Number, Girth, Graph Rank, Independence Number, Domination Number and graph nature (Acyclic, Tree, Complete, Eulerian, Hamiltonian and Regular) from left to right.

Table 5. Basic graph properties for disconnected constellations

Con.	E	N	CN	CI	VC	EC	C	G	GR	IN	DN	Acy	Tree	Cli.	Eul.	Ham.	Reg.
And	15	16	2	4	1	1	2	∞	15	9	6	T	T	F	F	F	F

Aqr	24	24	2	3	1	1	2	12	23	13	8	F	F	F	F	F	F
Cnc	5	6	2	3	1	1	2	∞	5	3	3	T	T	F	F	F	F
CVn	3	4	2	2	1	1	2	∞	3	2	2	T	T	F	F	F	F
CMi	1	2	2	1	1	1	2	∞	1	1	1	T	T	T	F	T	T
Cap	11	11	2	3	1	1	2	10	10	6	4	F	F	F	F	F	F
Car	10	11	2	2	1	1	2	∞	10	6	4	T	T	F	F	F	F
Cas	4	5	2	2	1	1	2	∞	4	3	2	T	T	F	F	F	F
Col	5	6	2	3	1	1	2	∞	5	3	3	T	T	F	F	F	F
Com	2	3	2	2	1	1	2	∞	2	2	1	T	T	F	F	F	F
Cru	2	4	2	1	0	0	2	∞	2	2	2	T	F	F	F	F	T
Dra	15	15	2	3	1	1	2	4	14	8	5	F	F	F	F	F	F
Eri	32	32	2	2	1	1	2	∞	31	16	11	T	T	F	F	F	F
Gem	17	18	2	4	1	1	2	∞	17	11	7	T	T	F	F	F	F
Gru	11	11	2	3	1	1	2	6	10	6	4	F	F	F	F	F	F
Her	19	19	2	3	1	1	2	4	18	10	6	F	F	F	F	F	F
Hya	20	20	2	3	1	1	2	4	19	10	7	F	F	F	F	F	F
Hyi	5	6	2	2	1	1	2	∞	5	3	2	T	T	F	F	F	F
Mon	6	7	2	3	1	1	2	∞	6	4	3	T	T	F	F	F	F
Oct	5	5	2	3	1	1	2	4	4	3	2	F	F	F	F	F	F
Oph	10	10	2	3	1	1	2	8	9	5	4	F	F	F	F	F	F
Ori	21	20	2	3	1	1	2	6	19	10	7	F	F	F	F	F	F
Peg	13	13	2	4	1	1	2	4	12	7	5	F	F	F	F	F	F
Phe	13	13	2	3	1	1	2	6	12	7	4	F	F	F	F	F	F
Pic	2	3	2	2	1	1	2	∞	2	2	1	T	T	F	F	F	F
Pyx	2	3	2	2	1	1	2	∞	2	2	1	T	T	F	F	F	F
Ret	6	6	2	2	2	2	2	6	5	3	2	F	F	F	T	T	T
Sge	3	4	2	3	1	1	2	∞	3	3	1	T	T	F	F	F	F
Sco	17	18	2	3	1	1	2	∞	17	9	7	T	T	F	F	F	F
Ser	11	11	2	3	1	1	2	4	10	6	4	F	F	F	F	F	F
Tau	13	14	2	3	1	1	2	∞	13	7	5	T	T	F	F	F	F
TrA	4	4	2	2	2	2	2	4	3	2	2	F	F	T	T	T	T
Tuc	6	6	2	2	2	2	2	6	5	3	2	F	F	F	T	T	T
Vir	14	14	2	4	1	1	2	4	13	7	6	F	F	F	F	F	F
Vul	4	5	2	2	1	1	2	∞	4	3	2	T	T	F	F	F	F
Aur	9	8	3	4	1	1	3	3	7	4	3	F	F	F	F	F	F
Boo	13	12	3	4	1	1	3	3	11	6	4	F	F	F	F	F	F
Cma	9	9	3	3	1	1	3	3	8	4	4	F	F	F	F	F	F
Cep	11	10	3	3	1	1	3	3	9	5	3	F	F	F	F	F	F
Cet	15	14	3	4	1	1	3	3	13	7	5	F	F	F	F	F	F
Cyg	12	11	3	4	1	1	2	5	10	6	4	F	F	F	F	F	F
Dor	7	6	3	4	1	1	3	3	5	3	2	F	F	F	F	F	F
Leo	16	15	3	5	1	1	2	4	14	8	6	F	F	F	F	F	F
Lep	13	11	3	4	1	1	2	4	10	6	3	F	F	F	F	F	F
Lib	6	6	3	3	1	1	3	3	5	3	2	F	F	F	F	F	F
Lup	10	9	3	4	1	1	3	3	8	4	3	F	F	F	F	F	F
Lyr	7	6	3	4	1	2	3	3	5	3	2	F	F	F	T	F	F

Nor	5	5	3	3	2	2	2	5	4	2	2	F	F	F	T	T	T
Per	17	16	3	4	1	1	2	5	15	9	5	F	F	F	F	F	F
Psc	17	16	3	3	1	1	3	3	15	7	5	F	F	F	F	F	F
PsA	10	9	3	4	1	2	3	3	8	4	3	F	F	F	T	F	F
Pup	9	9	3	3	1	1	2	5	8	4	3	F	F	F	F	F	F
Sgr	22	19	3	4	1	1	3	3	18	10	6	F	F	F	F	F	F
Scu	5	5	3	3	2	2	2	5	4	4	2	F	F	F	T	T	T
Tri	3	3	3	3	2	2	3	3	2	1	1	F	F	F	T	T	T
Uma	22	20	3	4	1	1	3	3	19	10	6	F	F	F	F	F	F

The Table 6 exhibits the network properties for disconnected constellation graphs. Diameter, Radius, Average Path Length, Density, Modularity, No. of Communities, Average Clustering Coefficient and No. of Triangles, from left to right of the table. Eridanus graph can be identified as the disconnected constellation graph with largest connected component while the Canis Minor is the smallest. Triangulum graph shows the highest clustering coefficient because of its complete nature.

Table 6. Network properties for disconnected constellation graphs

Name	D	R	AD	APL	Den.	Mod.	C	ACC	T
And	9	5	1.875	3.58	0.125	0.529	4	0	0
Aqr	13	7	2	5.217	0.087	0.579	5	0	0
Cnc	4	2	1.667	2.067	0.333	0.26	3	0	0
CVn	3	2	1.5	1.667	0.5	0	1	0	0
CMi	1	1	1	1	1	0	1	0	0
Cap	6	5	2	3	2	0.364	4	0	0
Car	10	5	1.818	4	0.182	0.465	3	0	0
Cas	4	2	1.6	2	0.4	0.219	2	0	0
Col	4	2	1.667	2.067	0.333	0.22	2	0	0
Com	2	1	1.333	1.333	0.667	0	1	0	0
Cru	∞	1	1	1	0.333	0.5	2	0	0
Dra	11	6	2	4.362	0.143	0.518	4	0	0
Eri	32	16	1.939	11.333	0.061	0.667	6	0	0
Gem	9	5	1.889	3.922	0.111	0.552	4	0	0
Gru	6	3	2	2.873	0.2	0.384	4	0	0
Her	11	6	2	4.503	0.111	0.536	4	0	0
Hya	18	9	2	6.821	0.105	0.576	4	0	0
Hyi	5	3	1.667	2.333	0.333	0.26	3	0	0
Mon	4	2	1.714	2.286	0.286	0.292	3	0	0
Oct	3	2	2	1.6	0.5	0.08	2	0	0
Oph	6	4	2	2.689	0.222	0.34	4	0	0
Ori	9	5	2.1	4.032	0.111	0.516	3	0	0
Peg	8	4	2	3.333	0.167	0.467	4	0	0
Phe	10	5	2	3.718	0.167	0.426	4	0	0
Pic	2	1	1.333	1.333	0.667	0	1	0	0
Pyx	2	1	1.333	1.333	0.667	0	1	0	0
Ret	3	3	2	1.8	0.4	0.167	3	0	0
Sge	2	1	1.5	1.5	0.5	0	1	0	0

Sco	15	8	1.889	5.993	0.111	0.555	4	0	0
Ser&	9	3	2	2.285	0.333	0.316	2	0	0
Tau	8	4	1.857	3.681	0.143	0.503	4	0	0
TrA	2	2	2	1.333	0.667	0	2	0	0
Tuc	3	3	2	1.8	0.4	0.167	3	0	0
Vir	6	3	2	3.088	0.154	0.523	3	0	0
Vul	4	2	1.6	2	0.4	0.219	2	0	0
Aur	4	2	2.25	2.035	0.321	0.278	3	0.214	1
Boo	6	3	2	2.863	0.167	0.405	5	0.233	1
CMa	5	3	2	2.528	0.25	0.46	3	0.389	1
Cep	5	3	2.2	2.467	0.244	0.355	3	0.185	1
Cet	8	4	2.143	3.626	0.165	0.433	2	0	0
Cyg	5	3	2.182	2.455	0.218	0.33	3	0	0
Dor	3	2	2.333	1.667	0.467	0.204	2	0.733	0
Leo	6	3	2.267	2.762	0.162	0.441	4	0	0
Lep	4	2	2.364	2.237	0.236	0.275	4	0	0
Lib	4	2	2	1.933	0.4	0.208	2	0.417	1
Lup	5	3	2.222	2.306	0.278	0.36	3	0.31	1
Lyr	3	2	2.333	1.667	0.467	0.204	2	0.361	1
Nor	2	2	2	1.5	0.5	0.08	2	0	0
Per	7	4	2.125	3.25	0.142	0.431	3	0	0
Psc	12	6	2.125	5.142	0.142	0.524	4	0.146	1
PsA	4	3	2.222	2.25	0.278	0.36	3	0.241	1
Pup	6	3	2	2.75	0.25	0.364	2	0	0
Sgr	9	5	2.316	3.994	0.129	0.486	5	0.143	2
Scu	2	2	2	1.5	0.5	0.08	2	0	0
Tri	1	1	2	1	1	0	1	1	1
UMa	10	5	2.2	4.232	0.116	0.537	5	0.094	1

Classification of Constellation Graphs

Based on the results that are given in tables, the following three classifications have been obtained.

The Figure 3 exhibits the classification done according to connected and disconnected nature of constellations. Further, they are classified by considering cyclic/acyclic nature and bipartite and non-bipartite nature. Also, cyclic graphs are divided by considering the number of vertices in the largest cycle of the graph.

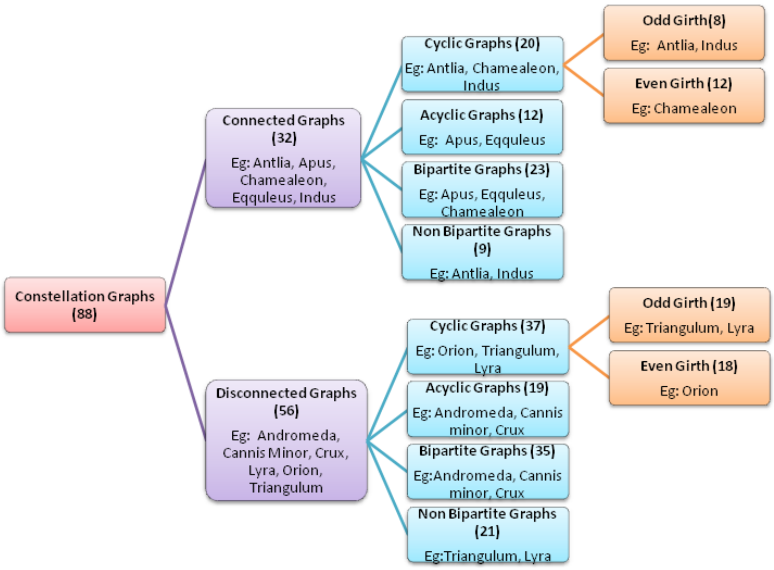

Figure 3. 1st Classification

Furthermore, Figure 4 shows the classification done by considering cyclic and acyclic nature of constellations. Further classification is done by analysing the basic graph properties obtained previously.

Not only that, Figure 5 depicts the classification done by considering the clique number of the graphs. Here, vertex connectivity (V.C) and edge connectivity (E.C) are also considered for the classification. Here, V.C1 indicates the group of graphs with vertex connectivity 1.

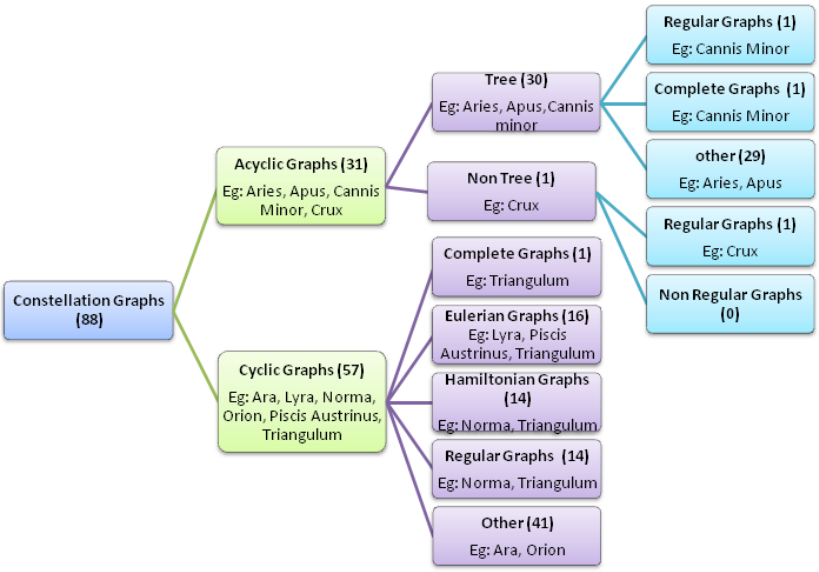

Figure 4. 2nd Classification

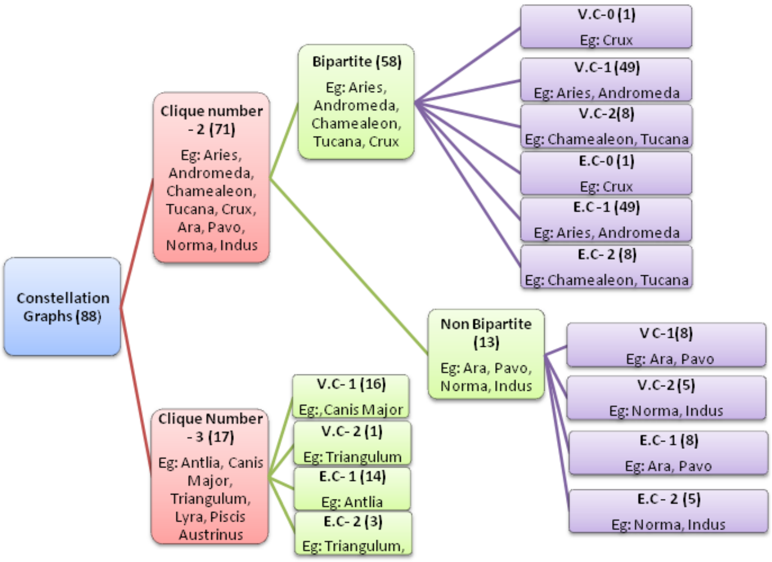

Figure 5. 3rd Classification

Few more classifications similar to these can be done by using obtained properties as well.

Conclusions

In this study basic graph and network properties of constellations are observed comprehensively. According to this analysis, numbers of constellations that satisfy those properties are given below. The numbers in brackets indicates that the how many graphs are holding the given property.

- All the constellation graphs are planar and simple.
- Chromatic number for every constellation graph is 2 (58) or 3 (30).
- For all the graphs, clique number is 2 (71) or 3 (17).
- Chromatic Index = {1 (2), 2 (27), 3 (38), 4 (19), 5 (1), 6 (1)}
- Vertex, Edge Connectivity = {0 (1), 1 (73), 2 (14)}
- There are 14 cycles, C_3 (1), C_4 (3), C_5 (5), C_6 (4) and C_8 (1).
- Girth is infinite (31) for acyclic graphs. For cyclic graphs, Girth = {3 (17), 4 (19), 5 (9), 6 (7), 7 (1), 8 (2), 10 (1), 12 (1)}
- Graph Rank = {1 (1), 2 (6), 3 (9), 4 (12), 5 (13), 6 (4), 7 (6), 8 (6), 9 (3), 10 (7), 11 (1), 12 (2), 13 (3), 14 (2), 15 (3), 17 (2), 18 (2), 19 (4), 23 (1), 31 (1) }
- Independence Number = {1 (2), 2 (18), 3 (21), 4 (14), 5 (5), 6 (7), 7 (7), 8 (2), 9 (3), 10 (5), 11 (2), 13 (1), 16 (1)}
- Domination Number = {1 (8), 2 (31), 3 (20), 4 (10), 5 (6), 6 (5), 7 (6), 8 (1), 11 (1)}
- Only 2 graphs can be identified as complete graphs which are in the form K_2 and K_3.
- There are 15 Hamiltonian graphs, 16 Eulerian graphs and 16 Regular graphs.
- Diameter= {1 (2), 2 (14), 3 (15), 4 (14), 5 (4), 6 (9), 7 (2), 8 (4), 9 (6), 10 (3), 11 (2), 12 (1), 13 (1), 15 (1), 18 (1), 31 (1), ∞ (1)}
- Radius= {1 (8), 2 (29), 3 (25), 4 (8), 5 (9), 6 (3), 7 (1), 8 (1), 9 (1), 16 (1)}
- Average Degree= {1 (2), 1.3 (4), 1.5 (5), 1.6 (3), 1.7 (7), 1.8 (5), 1.9 (5), 2 (39), 2.1 (5), 2.2 (7), 2.3 (5), 2.4 (1)}
- Average Degree= {1 (2), 1.3 (8), 1.5 (6), 1.6 (4), 1.7 (6), 1.8 (4), 1.9 (2), 2 (4), 2.1 (2), 2.2 (2), 2.3 (10), 2.4 (2), 2.5 (4), 2.6 (1), 2.7 (3), 2.8 (2), 2.9 (2), 3 (2), 3.1 (1), 3.3 (3), 3.6 (2), 3.7 (2), 3.9 (1), 4 (4) , 4.2, 4.4, 4.5, 5.1, 5.2, 6, 6.8, 11(1)}
- Density= {0.1 (14), 0.2 (18), 0.3 (21), 0.4 (30), 0.5(15), 0.7 (8), 1 (2)}
- Modularity= {0 (12), 0.1 (9), 0.2 (16), 0.3 (18), 0.4 (14), 0.5(13), 0.6 (5), 0.7 (1)}

• No. of Communities= {1 (8), 2 (28), 3 (19), 4 (13), 5 (4), 6(1)}

• No. of Triangles= {0 (73), 1 (11), 2 (1)}

• There are 20 Path graphs, 6 Star graphs, 33 1-Trees, 68 Pseudo trees, 23 Caterpillar trees, 77 Cactus graphs, 21 Lollipop graphs, 13 Tadpole graphs, 1 Fish graph, 5 Hyper cube graphs and 5 Ladder graphs.

The international Astronomical Union has defined the constellations by its border but not its pattern. So, these hypothetical patterns can be changed and differed from another pattern defined for the same constellation. Also, the obtained values can be differed according to the pattern we chose for the constellation. Moreover, these constellations graphs can be further analyzed with more graph theory properties and can be categorized in many ways in order to find more common properties for these constellation graphs. Then the new constellation theories can be defined by using these types of properties.

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