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On timeline of enhancing testing-capacity of COVID-19: A case study via an optimal replacement model

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ABSTRACT

Process of enhancing testing-capacity regarding COVID-19 is a topic of interest. This task of enhancing is constrained by socio-economic background of a country either in favorable or unfavorable ways. In this paper, we investigate timing of enhancing testing-capacity as an optimal problem, where the enhancement is quantified via number of tests as an instant measure and recovered portion as a long-term measure. The proposed work is structured analogous to an optimal machine replacement model based on a non-linear integral equation. Overall model is partially identifiable and compatible parameter estimations are carried out for a specific case study covering an early stage scenario. In addition, scenario development criteria on demand and effort for enhancing testing-capacity are introduced for predictions. In one numerical experiment, it is observed that frequency of enhancing testing-capacity starts decreasing after two increments indicating a favorable direction amidst effort constraints.

1. Introduction

COVID-19 has become a pandemic in a rapid phase causing almost every individual in the world a susceptible. Enhancing testing-capacity (ETC) is a key activity in mitigating this disease both in short and long run [1,2]. There are two types of tests available as viral test and antibody test. The viral test has more capability to reflect a prevailing situation since antibodies to infection may take 1–3 weeks to appear [3]. Real-time RT-PCR test is the viral test used in practice that detects nucleic acid from the coronavirus, SARS-CoV-2 [4]. This test facilitates early isolation and contact tracing, which are important aspects of controlling the spread [5]. Thus, detected positive cases are hospitalized and their close-contacts undergo quarantine (self-quarantine or at designated centers). It is emphasized that reports on travel history are more important than chest radiography since testing can be implemented on target groups as early as possible [6].

In any infectious disease, testing-based isolation remains effective only if infectiousness is identified before the onset of symptoms [7,8]. As per the evidence, COVID-19 shows a significant risk being transmitted before symptoms, unlike SARS, MERS and Ebola [9,10]. In the latter diseases, infectiousness comes into effect after symptoms or concurrently [11–13]. Therefore, it is always defensive if health authorities can optimally carry-out ETC to face such risky transmission. The proposed work is greatly motivated by the mixed outcome of success and struggles regarding testing-capacity of different local administrations and governments [14]. The original epicenter China had issued frequent guidelines on diagnosis showing the importance of testing [15]. World Health Organization (WHO) is also in a continuous mission of urging improvements in testing-capacity [16]. Countries worldwide have been taking preventive measures in general and in region-specific or community-specific manner [17]. Meanwhile, there is an emerging resurgence of the disease forcing for second-wave mitigation [18]. Further, there is always a dilemma whether to extend or relax control measures and if so, then in which level [19]. Under these circumstances, model-based scenario development for ETC would support decision making.

ETC can be of different procedures such as increasing number of tests per unit time, introducing random tests in addition to targeted tests, increasing test kit production and supply, improving quality of tests etc. In the literature, these procedures are referred along with different terminologies such as limit of detection, accessibility and acceptability of tests, coordinated testing, population-scale testing, testing technologies etc. [20,21]. Start

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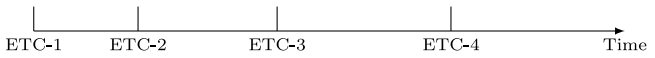

Fig. 1. Example: Timeline of ETC with four procedures ETC-1: Start of targeted tests, ETC-2: Increasing number of tests per unit time, ETC-3: Introducing random tests in addition to targeted tests, ETC-4: Increasing test kit production and supply.

of targeted tests is essentially the first ETC procedure. Many of the procedures basically hint an increase of number of tests and follow a timeline in days as in the example depicted in Fig. 1.

ETC is welcome by any healthcare administration against overwhelming conditions. However, the efforts rendered towards ETC may have socio-economic limitations arising from the availability of laboratory facilities, the capability of bearing the cost and preparedness with policy decisions and public compliance [21]. Thus, massive scale testing is not practical leaving the demand for well-coordinated testing strategies while catering the priorities [22]. Our work addresses optimal timing for ETC procedures, first acquiring characteristics of the initial phase of transmission and then proposing a prediction strategy. Such predictions are important for suppressing further outbreaks, resuming economic activities and ensuring herd immunity [23]. Modeling perspective of ETC requires feedback from ongoing testing capacity, quantification on positive cases and recovered cases [24]. Response of an ETC procedure should be quantified in comparison with a situation of such ETC had not been occurred. Under continuous transmission, overwhelming conditions alarm on whom to prioritize for testing and quarantine, when and how. Therefore, frequent ETC procedures suggests better prevention [25].

In mathematical terminology, the objective of maximizing the outcome of ETC is subject to effort constraints mentioned above. This becomes an optimal replacement-like scenario, where an extra cost is vested with any replacement. For instance, replacing older machines by new machines reflects such a scenario. However, that extra cost can be tallied with the gain of replacement which sets a routine for optimal context. As presented in [26,27] and followed by further studies in [28], the integral equation approach has vast applicability in machine replacement problems. Technological progress and deterioration over time are the key concerns forcing a replacement. In the sequel, we put forward a similar concept, where technological progress is resembled by *growing demand* for ETC. Such a demand would be expedited by scientific findings and learning by doing in practice. Next, the delay in detecting infected cases and risk of transmission within that delay cause a *deterioration* of any procedure of ETC over time. It is much of the same as any machine continues to deteriorate as time passes.

The optimization routine that we adopt is given in (1).

$$\int_{I_L} \text{Gain of ETC} = \text{Effort for ETC} \ ; \qquad I_L \subseteq [0, \infty) \tag{1}$$

Here, I_L stands for the time interval of an ETC procedure (i.e. from the start of an ETC procedure until the start of next procedure). As the numerical solution, we expect an optimal sequence of time periods $\{L_k, k = 1, 2, \ldots\}$ given by I_L.

This integral model brings the notion of tallying the gain of ETC with the effort putting in ETC, analogous to tallying the gain of machine replacement with the cost for that replacement. Thus, we hypothetically transform cost into effort with a view to give a fair quantification. In replacement studies, such a model is derived by maximizing product output while minimizing expenses of purchased machines [27,29]. The solution of the integral model provides optimal timing of replacements and in our application, the solution provides optimal timing of ETC procedures. In the integral model, several factors are combined such as the number of positive cases and their close-contacts reported in early transmission, frequency of tracing close-contacts, the delay from symptom onset to isolation and proportion of transmission before to symptoms. These epidemiological and surveillance measures are crucial in deciding feasibility of control [5,7,30]. We design parameters in a compatible way to include these measures into the model equation projected from (1).

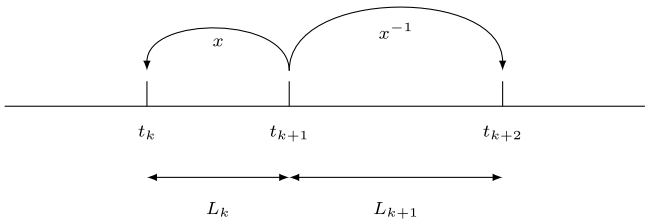

Fig. 2. Operating intervals.

As per the analysis done in [27,28], three scenarios can be developed as the rate of the growing demand for ETC equals/less than/greater than to the rate of putting effort in ETC. Latter two cases yield a recursive behavior of optimum $\{L_k, k = 1, 2, \ldots\}$, while L_k becomes constant under equal rates. We test the model for Sri Lanka, where well-documented ETC procedures are available. A feedback is provided to the model via a fuzzy operator combining two important aspects: number of PCR tests and recovered portion. The novelty of our approach lies on three pillars: new application of optimum replacement model, compatible parameter design and computational scheme for numerical solutions. Several statistics on predicted L_k and unobservable parameters are also provided.

2. Modeling framework

We first introduce the mechanism of timeline variation. It later plays the central role in model solution.

2.1. Time variable

We set $t_1 = 0$ as the starting time (day) of targeted tests (ETC-1) and a continuous function x such that $t_1 = x(t_2)$. Here, t_2 represents the day that the second ETC procedure (ETC-2) is announced. Thus, the function x relates the announcing time of two events: ETC-1 and ETC-2. Concurrently, x^{-1} (inverse function of x) becomes the decider of future timing as seen in $x^{-1}(t_1) = x^{-1}(x(t_2)) = t_2$. For analytical perspective we may assume the existence of x^{-1}. In general, our aim is to work on optimum *operating periods* $L_k, k = 1, 2, \ldots$, where $L_k = t_{k+1} - x(t_{k+1}) = t_{k+1} - t_k$. Note that t_k is the day that ETC-k is announced. There is a span of a few days for any ETC procedure to come into effect. However, as a simple approach, we start counting the duration of operating periods from the day after the announcing day of each procedure (open bracket to start with). In the sequel, we will relate the following consecutive operating intervals for $k = 1, 2, \ldots$.

$(t_k, x^{-1}(t_k)] = (t_k, t_{k+1}]$ — Operating interval of ETC-k with the interval length L_k

$(t_{k+1}, x^{-1}(t_{k+1})] = (t_{k+1}, t_{k+2}]$ — Operating interval of ETC-$(k+1)$ with the interval length L_{k+1}

The above two intervals are illustrated in Fig. 2. We can search time-point of next ETC procedure via x^{-1}. Later, we come across a convenient recursive formula for this.

2.2. Model equation — Efficiency function

The left-hand side of (1) is described via a function (b) called *healthcare efficiency*. The gain of ETC associates with the variation of that healthcare efficiency. Growing demand and deterioration introduced earlier come into effect here. As motivated by [26–28], we define $b(t, u)$ as the function of healthcare efficiency per unit time experienced at a current time u due to ETC procedure announced at time t. The measuring unit of this entity is redundant in the final evaluation of L_k, $k = 1, 2, \ldots$ as we later measure the effort in the right-hand side of (1) by the same efficiency unit. On the other hand, measuring a service sector efficiency such as in healthcare is harder than that of the manufacturing sector. Although the quantity of production can be easily transformed to an efficiency unit in the manufacturing sector, it is required to measure both tangible (e.g. number of medical staff, number of beds, capacity of labs etc.) and intangible components (e.g. patient satisfaction) in service sector [31].

To cater the left-hand side of (1), the total gain of ETC experienced during $(t, x^{-1}(t)]$ is modeled by $B(t)$ in (2). We recall that ETC procedure announced at t has an operating period until $x^{-1}(t)$, hence the limits of the integration.

$$B(t) = \int_t^{x^{-1}(t)} [b(t, u) - b(x(u), u)]\, du \tag{2}$$

Note that $b(x(u), u)$ stands for the efficiency if previous ETC procedure occurs at $x(u)$, a time before t. In fact, it suggests that current ETC procedure occurs now at u instead of earlier t and hence we must rely on previous one for the prevailing efficiency. This scenario can be further convinced via x^{-1}. Note that $x^{-1}(x(u)) = u$ and $x^{-1}(t) \geq u$ suggest that recent ETC at $t(> x(u))$ directs next ETC (at $x^{-1}(t)$) delayed than u. In optimal replacement models, the usual design of $b(t, u)$ has an exponential form, where parameters are designed to capture technological progress and deterioration [27,28]. That classical $b(t, u)$ is of the form;

$$b(t, u) = b_0 e^{c_b t - c_d(u-t)}. \tag{3}$$

Here, b_0 stands for initial efficiency. The term $e^{c_b t}$ is for the growing demand for ETC and $c_b(> 0)$ is named as *demanding rate*. Next, $e^{-c_d(u-t)}$ represents deterioration due to delay in detecting infected cases and risk of transmission within that delay. Here, we name $c_d(> 0)$ as *deterioration rate*. Thus, the efficiency is less when c_d is higher. Furthermore, it decreases when the gap $u - t$ increases. Next section presents how to cope with c_b and c_d.

2.3. Parameter design of efficiency function

From a considerable number of factors affecting COVID-19 transmission, we recognize two to associate with c_b; ratio of secondary positive cases (S_0) and proportion (ρ) of close-contacts that are traced per unit time [day^{-1}]. Here, by a secondary case, we mean a positive case who is infected from a positive case diagnosed earlier. S_0 is estimated for a *preliminary period* (span of a few days or weeks from the start of testing) as in (4). Selection of a preliminary period is twofold: availability of data and reasonable span to extract early epidemiological situation.

$$S_0 = \frac{\text{Number of secondary positive cases}}{\text{Number of all positive cases}}. \tag{4}$$

It is clear that any healthcare system is forced towards frequent ETC procedures when S_0 and ρ are high. This resembles a situation of technological progress forcing machine replacement. We assign that S_0 and ρ govern c_b in a proportionate way as in (5). Here, α is the proportional constant, which reflects the level of taking (lightly/strongly) the message shown by $S_0 \times \rho$ by a country or a health administration. Enumeration of α is done by a computational scheme (Section 3.3) once the whole framework is designed.

$$c_b = \alpha \times S_0 \times \rho \tag{5}$$

We consider two factors to associate with c_d, namely delay from symptom onset to isolation (by τ) and rate of transmission before symptom onset per unit time (by ϕ). These two parameters are also compiled in a proportionate way as in (6). Here, β is the proportional constant, which indicates how far the deterioration is taken into consideration.

$$c_d = \beta \times \tau \times \phi \tag{6}$$

Here, $\tau \in (0, 1]$ is designed as in (7).

$$\tau = \frac{\text{Number of days delayed from symptoms to isolation}}{\text{Maximum possible days of delay}} \tag{7}$$

The denominator term can be taken as the mean period from symptoms to severe conditions, assuming a person with severe conditions surely be hospitalized without further delay. Thus, τ captures compliance of the general public to engage with isolation.

The parameter ϕ would be taken the reciprocal of the mean incubation period since longer the period, symptoms are delayed. Then the daily show up of being asymptomatic reduces when the incubation period is longer. Asymptomatic individuals can transmit the virus during this incubation period [7,8]. Enumeration of β again comes in Section 3.3.

Later we proceed with the parameters S_0, ρ, τ and ϕ assuming that they are reasonably abide by a random generation of proportional constants α and β to reflect healthcare situation. Thus, above four parameters act as identifiable ingredients of the rates c_b and c_d, while proportional constants tally unidentifiable characteristics. This context finally leaves our model partially identifiable [32].

2.4. Model equation — Effort function

Now for an optimal outcome as described with (1), we should tally the effort for ETC procedure at t with the total gain $B(t)$ in (2). By defining a function $p(t)$ for the effort, we reach the following non-linear Volterra integral equation.

$$\int_t^{x^{-1}(t)} [b(t, u) - b(x(u), u)]\, du = p(t) \tag{8}$$

The *effort function* $p(t)$ is also designed in exponential way as;

$$p(t) = p_0 e^{c_p t}. \tag{9}$$

Here, p_0 stands for initial effort and $p(t)$ is tolerated by the capacity of putting the effort for ETC. The parameter $c_p(> 0)$ should absorb that effect, which we name as *effort rate*.

Now by substituting from (3) and (9), model equation (8) becomes;

$$b_0 \int_t^{x^{-1}(t)} \left[e^{c_b t - c_d(u-t)} - e^{c_b x(u) - c_d(u - x(u))}\right] du = p_0 e^{c_p t}, \tag{10}$$

which owes the simplified version;

$$b_0 \int_t^{x^{-1}(t)} \left[e^{(c_b + c_d)t - c_d u} - e^{(c_b + c_d)x(u) - c_d u}\right] du = p_0 e^{c_p t}. \tag{11}$$

In solving (10), it is redundant to know b_0 and p_0 separately. As a rough initializing, we take the following from the preliminary period suggested earlier for evaluating S_0.

$$\frac{p_0}{b_0} = \frac{\text{Number of close-contacts}}{\text{Number of all positive cases}} = \mu \text{ (say)} \tag{12}$$

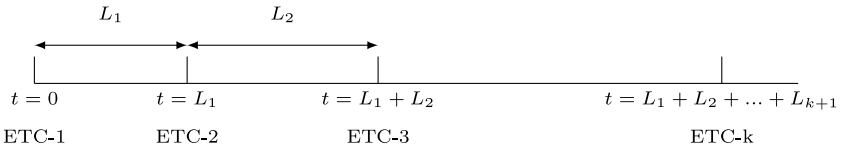

Fig. 3. Timeline of ETC and operating periods.

As simulated by Hellewell et al. [5], the number of initial cases always influence the final outcome. Thus, (12) facilitates it while featuring the strength of preliminary testing via the number of all positive cases (denominator in (12)) and the strength of initial effort via the number of close-contacts (numerator in (12)).

Now we can present the model equation as;

$$\int_t^{x^{-1}(t)} \left[e^{(c_b+c_d)t-c_d u} - e^{(c_b+c_d)x(u)-c_d u}\right] du = \mu e^{c_p t}. \tag{13}$$

2.5. Discounting effect

In machine replacement, future expenditure which is set now is less worthy due to depreciation of monetary value. Therefore, a discounting effect must be incorporated, which is usually done via an exponentially decaying term e^{-rt}; $r > 0$ [26]. This effect should also be imposed on the gain of replacement since the gain should be under-estimated as our expenditure is less worthy. We apply a similar principle for ETC assuming announced ETC procedures are less worthy in practice as time passes. Then, the new version to continue is;

$$\int_t^{x^{-1}(t)} e^{-ru}\left[e^{(c_b+c_d)t-c_d u} - e^{(c_b+c_d)x(u)-c_d u}\right] du = e^{-rt}\left(\mu e^{c_p t}\right) = \mu e^{(c_p-r)t}. \tag{14}$$

In our context, we name this discounting rate r as *friction-in-practice* since higher r yields rapid loss of ETC benefit. It may take the cumulative influence of imported cases, asymptomatic nature in bigger clusters, relaxing social distancing and hygiene practices etc. The parameter r is also unidentifiable and will be decided in compromise with α and β.

3. Experimental results

3.1. Operating periods and scenario development

Differentiation of (14) with respect to t leads to (15) assuming differentiability of x^{-1} (see Appendix and [27]).

$$(r+c_d)e^{-(c_b+c_d)L(t)} - (c_b+c_d)e^{-(r+c_d)L(x^{-1}(t))} = r - c_b - \mu(r+c_d)(r-c_p)e^{(c_p-c_b)t}. \tag{15}$$

Here, $L(t)$ is the operating period of an ETC procedure implemented at time $x(t)$ (i.e. $L(t) = t - x(t)$), while $L(x^{-1}(t))$ is the operating period of ETC procedure implemented at time t (i.e. $L(x^{-1}(t)) = x^{-1}(t) - x(x^{-1}(t)) = x^{-1}(t) - t$). Thus, (15) yields a recursive formula that relates two consecutive operating periods $L(t)$ and $L(x^{-1}(t))$.

The discrete structure of the above recursion is given in (16). This is in accordance with the notations introduced in Section 2.1. Note that t is updated in the additive manner for operating periods as depicted in Fig. 3.

$$(r+c_d)e^{-(c_b+c_d)L_k} - (c_b+c_d)e^{-(r+c_d)L_{k+1}} = r - c_b - \mu(r+c_d)(r-c_p)e^{(c_p-c_b)(L_1+L_2+\cdots+L_k)} \tag{16}$$

From (16), the explicit version of L_{k+1} in terms of previous L_k becomes;

$$L_{k+1} = \frac{-1}{r+c_d} \times \ln\left[\frac{(r+c_d)e^{-(c_b+c_d)L_k} - r + c_b + \mu(r+c_d)(r-c_p)e^{(c_p-c_b)(L_1+L_2+\cdots+L_k)}}{c_b+c_d}\right]. \tag{17}$$

Table 1
Scenario development via parameters c_b and c_p.

Scenario	I (equally treated)	II (under-treated)	III (over-treated)
Quantification	$c_p = c_b$ $(\gamma = 1)$	$c_p < c_b$ $(\gamma < 1)$	$c_p > c_b$ $(\gamma > 1)$
Risk	Moderate	High	Low

One can evaluate L_{k+1} under three scenarios, where demanding rate c_b is equally treated/ under-treated/ over-treated by the effort rate c_p. These are shown in Table 1 with general risk to healthcare. We set $c_p = \gamma c_b$ in further incorporation and name γ as *effort-to-demand ratio*.

We interchangeably use (16) or (17) in consequent calculations. Parameter γ needs to be estimated via feedback of healthcare measures. For this we consider two aspects in tandem. First, the number of PCR tests as a logistic measure (data source: [33]) and secondly the recovered portion as an epidemiological measure (data source: [34]). We estimate γ via fuzzy membership functions, with the view of bringing a combined effect. First, we define a fuzzy set to cater the number of PCR tests as follows.

$$A = \{(t_n, U_a(t_n)); t_n \in T\} \text{ where} \tag{18}$$
$$U_a(t_n) := \frac{\text{Moving average of daily PCR tests at } t_n}{\text{Maximum of above moving averages}}$$

Here, moving average covers a week centering a day (t_n) to smooth daily variations. T represents the period of concern since ETC-1.

Next, we define a fuzzy set B representing recovered portion. Here also, a week around a day is covered by moving average.

$$B = \{(t_n, U_b(t_n)); t_n \in T\} \text{ where} \tag{19}$$
$$U_b(t_n) := \frac{\text{Moving average of } \frac{\text{Total recovered cases}}{\text{Total positive cases}} \text{ at } t_n}{\text{Maximum of above moving averages}}$$

Clearly, $U_a(t), U_b(t) \in [0, 1]$ and higher membership values indicate better effort. In order to estimate γ, we combine these two membership functions U_a and U_b via Hamacher t-norm operator

$$U_a(t_n) * U_b(t_n) := \frac{U_a(t_n)U_b(t_n)}{U_a(t_n) + U_b(t_n) - U_a(t_n)U_b(t_n)}. \tag{20}$$

Note that $U_a(t_n) * U_b(t_n)$ increases when $U_a(t_n)$ or $U_b(t_n)$ increases for a fixed $U_b(t_n) \neq 0$ or $U_a(t_n) \neq 0$ respectively. It suits for the general behavior of the combined effect as an increase of either memberships (i.e. for PCR tests and recovered portion) indicates a better effort. Since we model γ around 1, $U_a * U_b \in [0, 1]$ is shifted to [0.5, 1.5] by adding 0.5, i.e. we take

$$\gamma \text{ at } t_n := U_a(t_n) * U_b(t_n) + 0.5. \tag{21}$$

By this shift, we assume that average level 0.5 in [0, 1] reflects the moderate level of risk ($\gamma = 1$). Further calculation procedure of γ can be seen in Section 3.3.

3.2. Estimation of identifiable parameters

Now, we estimate parameters using data and observations from Sri Lanka. March 11, 2020, is the date that the first Sri

Table 2
Parameter estimation — Sri Lanka.

Parameter	Estimation	Source
S_0	0.3039	Data — [37]
ρ	0.1286	Based on observation[a]
τ	0.25	Based on literature [38] and observation[b]
ϕ	0.1923[c]	Based on literature [39]
$\frac{p_0}{b_0}(=\mu)$	19.78[d]	Data — [37]

[a]Assuming 90% of close-contacts traced within a week due to extensive surveillance with the participation of health and military officials.
[b]Assuming a delay of 2 days from symptoms to isolation. Maximum delay = 8 days (mean period from symptoms to severe conditions [38]).
[c]Taken as the reciprocal of the incubation period. Incubation period = 5.2 days [39].
[d]Repetition of close-contacts counted.

Lankan case reported [35]. Prior to this, only one imported case, a Chinese tourist from Hubei reported on January 27, 2020, and recovered on February 19, 2020. Since March 11, positive cases has been on the rise [34,36]. We take March 11 as the start of targeted tests and hence the ETC-1. Then, the counting for operating periods begins from March 12. Certain specific attributes can be retrieved by contact mapping of Covid-19 diagnosed cases of Sri Lanka issued on March 27, 2020, available in [37]. Based on this case-specific data, we consider a 15-day preliminary period from March 11 to 26. This data source contains how secondary positive cases are connected with earlier positive cases. In addition, a number of close-contacts for each positive case is also available.

Table 2 summarizes all parameter estimations. Parameters catering general aspects are taken from literature and observation. Preliminary description on S_0, ρ, τ and ϕ is available in Section 2.3 and that of μ contains in Section 2.4.

According to those case-specific estimations, an influences on the demanding rate c_b by a change in ρ is more than that of S_0 since $S_0 > \rho$. Similarly, an influences on the deterioration rate c_d by a change in ϕ is more than that of τ since $\tau > \phi$.

By plugging the estimated values in Table 2, general recursive formula in (16) becomes (22).

$$(r+0.05\beta)e^{-(0.04\alpha+0.05\beta)L_k} - (0.04\alpha+0.05\beta)e^{-(r+0.05\beta)L_{k+1}} = r - 0.04\alpha - 19.78(r+0.05\beta)(r-0.04\alpha\gamma)e^{0.04\alpha(\gamma-1)(L_1+L_2+\cdots+L_k)} \tag{22}$$

Next, we search for compatible proportional constants α and β, and friction-in-practice rate r subject to identification of first three operating periods $L_k; k = 1, 2, 3$ as an optimal context. Sri Lankan health authorities issued recommendations to expand testing and diagnosis on April 07, 2020 [40]. This is considered to be the second ETC procedure (ETC-2). Next as the third ETC procedure (ETC-3), we have a notification on introducing random tests in addition to targeted tests on April 22, 2020, [41,42]. The fourth ETC procedure (ETC-4) is considered to be on April 30, 2020, as per the directive of Presidential Secretariat — Sri Lanka [43]. It is about producing test kits locally to strengthen the testing capacity. These procedures direct us to take $L_1 = 27$ days (from Mar 12 to Apr 07), $L_2 = 15$ days (from Apr 8 to Apr 22) and $L_3 = 8$ days (from Apr 23 to Apr 30), which are shown in Fig. 4.

3.3. Estimation of unidentifiable parameters

We follow the computational scheme below to estimate α, β and r.

Step 1: Consider the first two equations in (22) given below (i.e. equations with $k = 1$ and $k = 2$). γ values along with corresponding membership values are depicted in Fig. 5. We take $\gamma_1 := \gamma$ at $t_n = 27$ and $\gamma_2 := \gamma$ at $t_n = 42$. Note that $t_n = 27$ is the time point that connects operating periods of ETC-1 and ETC-2. Similarly, $t_n = 42$ connects operating periods of ETC-2 and ETC-3.

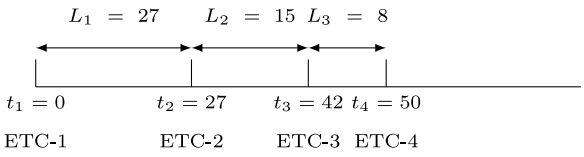

Fig. 4. Timeline of observed ETC procedures — Sri Lanka.

$$(r+0.05\beta)e^{-(0.04\alpha+0.05\beta)L_1} - (0.04\alpha+0.05\beta)e^{-(r+0.05\beta)L_2} = r - 0.04\alpha - 19.78(r+0.05\beta)(r-0.04\alpha\gamma_1)e^{0.04\alpha(\gamma_1-1)L_1} \tag{23}$$

$$(r+0.05\beta)e^{-(0.04\alpha+0.05\beta)L_2} - (0.04\alpha+0.05\beta)e^{-(r+0.05\beta)L_3} = r - 0.04\alpha - 19.78(r+0.05\beta)(r-0.04\alpha\gamma_2)e^{0.04\alpha(\gamma_2-1)(L_1+L_2)} \tag{24}$$

Step 2: Substitute $L_1 = 27$, $L_2 = 15$, $L_3 = 8$ along with $\gamma_1 = 0.7$ and $\gamma_2 = 1.02$ (taken from Fig. 5) into (23) and (24). Thereafter, solve these two equations for positive α, β and r by MATLAB solver fsolve giving initial guesses.

Step 3: Determine L_2 by substituting $L_1 = 27$ in (17). Next, substitute resulting L_2 along with $L_1 = 27$ in (17) to determine L_3.

Step 4: See the similarity of the answers in Step 3 with $L_2 = 15$ and $L_3 = 8$ at least as rounded to the nearest integer. If they do not tally, we continue Step 2 with different initial guesses generated randomly until we get $L_2 = 15$ and $L_3 = 8$ in Step 3. For α and β, we systematically update the guesses as to pick random numbers in $(n-0.5, n+0.5); n = 1, 2, \ldots$. The initial guess of r is set as zero while expecting positive values for the estimated r.

The above scheme computes α, β and r values that mechanically provide $L_1 = 27$, $L_2 = 15$ and $L_3 = 8$ correctly. It provides different combinations of α, β and r for different initial guesses due to non-linear and under-determined nature of the system of (23) and (24). In addition, rounding of L_{k+1} to the nearest integer also supports a random effect on α, β and r. In the sense of healthcare, this unidentifiable nature is acceptable as different levels of demand rate (c_b) and deterioration (c_d) may compromise same outcome subject to the variation of discounting effect (via r) too.

3.4. Statistics of parameters

Here we present how likely the parameters α, β and r behave in their random generation by simulation runs. In classical calibration approaches such as least-square minimization, maximum likelihood estimation and Bayesian inference, one can be confident about estimated parameters when higher number of data points are available. However, we deal with only three response values L_1, L_2 and L_3 in our computational scheme, where an algebraic satisfaction of explanatory parameters (α, β and r) prevails with many possible combinations. Therefore, it is worth to observe their own statistics (mean, median, skewness) and correlation between parameters before proceeding to further analysis and predictions. Then we have the distributional aspect of parameters structured by an algebraic context.

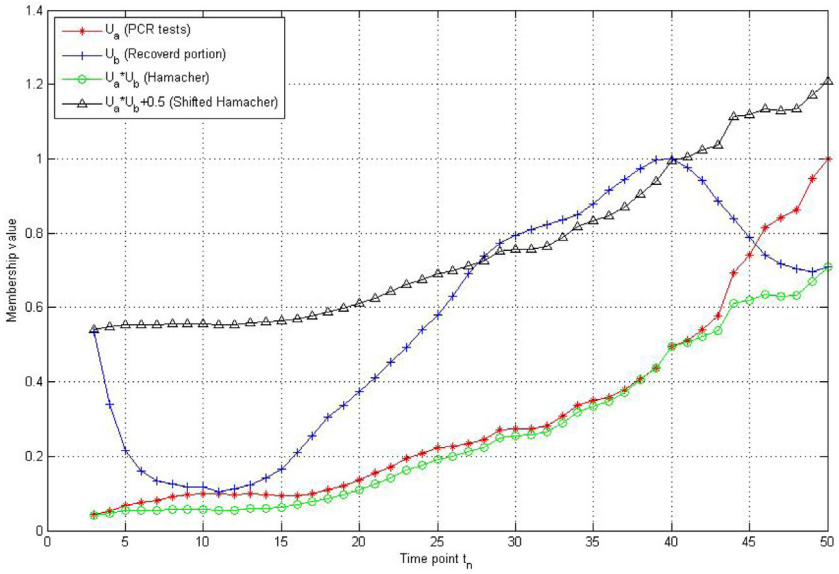

Fig. 5. Plot of γ.

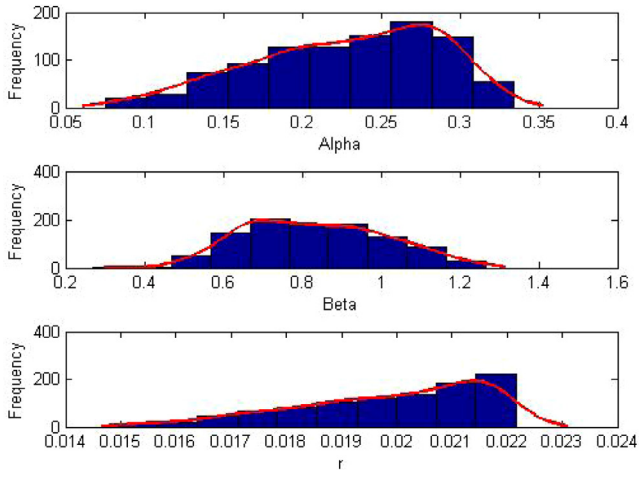

Fig. 6. Histograms (bars) and kernel-smoothing fit (curves) of α, β and r.

Table 3
Statistics of parameters α, β and r.

Parameter	Skewness	Median (Bootstrap CI)	Mean (Bootstrap CI)	Standard deviation
α	−0.4494	0.2369 (0.2317, 0.2414)	0.2280 (0.2243, 0.2316)	0.0576
β	0.1674	0.8180 (0.8044, 0.8347)	0.8329 (0.8219, 0.8437)	0.1770
r	−0.6941	0.0203 (0.0201, 0.0204)	0.0199 (0.0198, 0.0200)	0.0017

We carry out a trial with 1000 simulation runs and statistics are shown in Table 3. Mean and median are followed by bootstrap confidence intervals (within brackets) as a supportive measure. We opt to bootstrap since actual distributions are not known [44, 45]. We use 10,000 resamples for better accuracy. Histograms with nonparametric kernel-smoothing fit are displayed in Fig. 6.

To observe statistical relationship of parameters, Pearson correlation coefficient (R) is evaluated as $R(\alpha, \beta) = -0.9951$, $R(\alpha, r) = 0.9948$ and $R(\beta, r) = -0.9798$. As α and β are negatively correlated, proportionality in c_b and c_d behaves in opposite directions. Parameter r is positively correlated with α. This can be expected as higher r compromises demand set by higher α. A negative correlation is seen between β and r to negotiate deterioration.

3.5. Prediction strategy

For the above 1000 simulation runs, L_4 can be evaluated explicitly by (25), which is the case of $k = 3$ in (17). Here, $c_p = \gamma_3 c_b$, where $\gamma_3 = 1.21$ connecting the operating periods of ETC-3 and ETC-4 (see Fig. 5).

$$L_4 = \frac{-1}{r + c_d} \times \ln\left[\frac{(r + c_d)e^{-(c_b + c_d)L_3} - r + c_b + \mu(r + c_d)(r - c_p)e^{(c_p - c_b)(L_1 + L_2 + L_3)}}{c_b + c_d}\right]. \quad (25)$$

We keep α and β fixed for each simulation run and adjust r into r_{new} as in (26).

$$r\left(\frac{\gamma_1 + \gamma_2}{2}\right) = r_{\text{new}}\left(\frac{\gamma_2 + \gamma_3}{2}\right) \quad (26)$$

Basic routine of (26) is to fix the multiplication of friction-in-practice (r) and effort-to-demand (γ), which behave contrastingly in practice. Effect of γ is taken as the average of two consecutive cases to compromise the effect of combined operating periods. This tactic does not eliminate random-effect of our model. We assume the proportionality given by α and β are time-invariant showing healthcare situation that naturally prevails for a country (here Sri Lanka) in a pandemic. By fixing strongly correlated parameters with r, now the new version r_{new} acquires more explanatory ability ($R(\alpha, r) \approx 1$ and $R(\beta, r) \approx -1$). Finally, this technique reduces the degree of freedom providing an approximate algebraic manipulation to our original scheme in Section 3.3.

The distribution of predicted L_4 values is shown in Fig. 7 corresponding to the earlier 1000 simulation runs. Statistics of L_4 are given in Table 4, where $L_4 \approx 5.5$. This relatively short operating period (< 1 week) suggests that ETC cannot be ignored immediately though the feedback shown by γ is improved.

We continue the prediction for another two periods L_5 and L_6 with the same r_{new} used for L_4. Therefore, the feedback provided

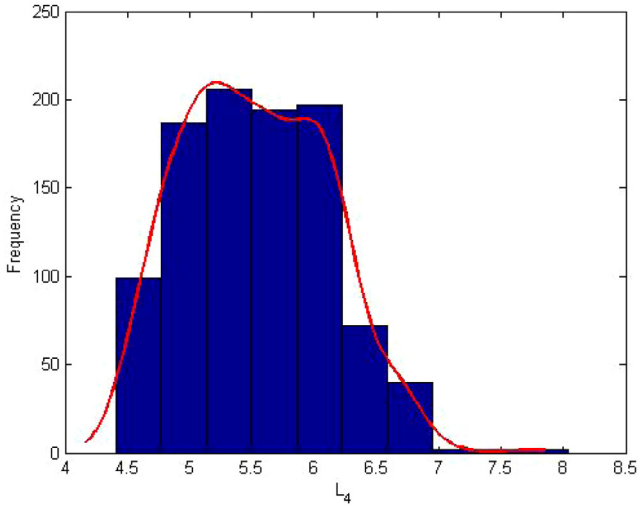

Fig. 7. Histograms (bars) and kernel-smoothing fit (curves) of L_4.

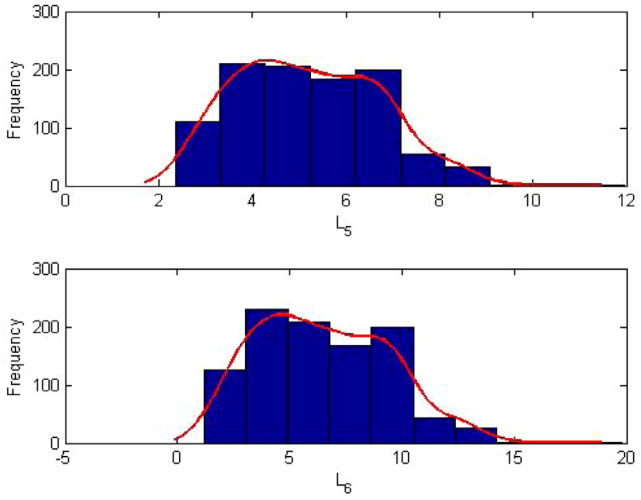

Fig. 8. Histograms (bars) and kernel-smoothing fit (curves) of L_5 and L_6.

Table 4
Statistics of operating periods L_4, L_5 and L_6.

Operating period	Skewness	Median (Bootstrap CI)	Mean (Bootstrap CI)	Standard deviation
L_4	0.3278	5.5114 (5.4561, 5.5568)	5.5299 (5.4943, 5.5662)	0.5806
L_5	0.3967	5.1275 (4.9827, 5.2481)	5.2016 (5.1089, 5.2978)	1.5238
L_6	0.4656	6.3078 (5.9896, 6.5351)	6.5056 (6.3212, 6.6911)	2.9494

by γ remains the same and we consider this outcome as a threshold to decide other alterations of r_{new}. Statistics of L_5 and L_6 are included in Table 4. Fig. 8 shows their distributions.

3.6. Sensitivity analysis of L_{k+1}

This section is devoted to sensitivity analysis of α and β in predicting operating period L_{k+1} by setting approximate conditions (i.e. $\gamma = 1.21$ and mean/median $r_{\text{new}} \approx 0.016$ from above 1000 simulation runs). This is to roughly recognize under which circumstance (i.e. α or β is more sensitive) that we should understand the situation. We perturb α and β by a small fraction $\varepsilon > 0$. By the Taylor's theorem for the expansion of L_{k+1} around (α, β) by a fraction ε, we have;

$$L_\alpha \equiv L_{k+1}(\alpha + \varepsilon\alpha, \beta) = L_{k+1}(\alpha, \beta) + \varepsilon\alpha \frac{\partial}{\partial\alpha} L_{k+1}(\alpha, \beta) + \mathcal{O}\left((\varepsilon\alpha)^2\right) \tag{27}$$

and

$$L_\beta \equiv L_{k+1}(\alpha, \beta + \varepsilon\beta) = L_{k+1}(\alpha, \beta) + \varepsilon\beta \frac{\partial}{\partial\beta} L_{k+1}(\alpha, \beta) + \mathcal{O}\left((\varepsilon\beta)^2\right). \tag{28}$$

We perturb α and β by a fraction ε (i.e. $\varepsilon\alpha$ and $\varepsilon\beta$) instead of a shift by ε (i.e. $\alpha + \varepsilon$ and $\beta + \varepsilon$). This is to observe relative changes since α and β associate with different contexts as t (via c_b) and $u - t$ (via c_d) respectively (see (3)).

The terms $\mathcal{O}\left((\varepsilon\alpha)^2\right)$ and $\mathcal{O}\left((\varepsilon\beta)^2\right)$ vanish for small ε. For an estimated combination of (α, β), we can claim that α is more sensitive than β, if $|L_\alpha| > |L_\beta|$, i.e. if $\left|\alpha \frac{\partial}{\partial\alpha} L_{k+1}(\alpha, \beta)\right| > \left|\beta \frac{\partial}{\partial\beta} L_{k+1}(\alpha, \beta)\right|$. The inequality alters to claim the reverse: β is more sensitive than α. It is possible to locate regions in a $\alpha\beta$-plane that indicate $|L_\alpha| < |L_\beta|$ and $|L_\alpha| > |L_\beta|$. An illustration is given in Fig. 9. Bootstrap confidence intervals of both median and mean of α and β lie on gray area suggesting α is more sensitive. Thus, we would see subsequent predictions under a circumstance that demand is more sensitive than deterioration.

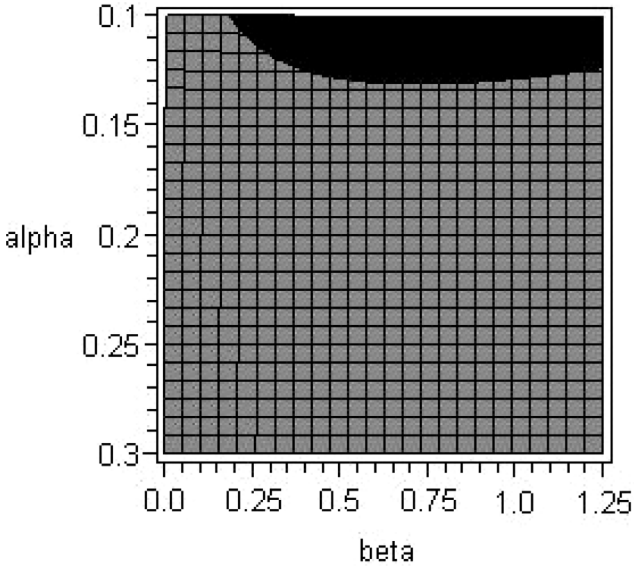

Fig. 9. Regions of sensitivity (Gray – α is more sensitive, Black – β is more sensitive).

4. Discussion and conclusion

Extracting every possible detail of the spread and mitigation of COVID-19 is a timely requirement. In this work, we propose a mechanism to investigate optimal timing for enhancing testing capacity (ETC). The key role is played by an integral model, which is frequently seen in optimal machine replacement [27]. We fetch it to a model that yields optimal timing of ETC procedures, where the gain of ETC is compromised with the effort to carry out ETC. Number of PCR tests acts as a quantifier of ETC. In addition, recovered portion passes a feedback into the proposed model.

Two types of parameters (identifiable and unidentifiable) are used directing the model partially identifiable. Compiling the

ratio of secondary positive cases (S_0) and proportion of close-contacts traced per unit time (ρ), we formulate the demanding rate $c_b = \alpha \times S_0 \times \rho$. The deterioration rate is formulated as $c_d = \beta \times \tau \times \phi$, where τ represents delay from symptom onset to isolation and ϕ represents rate of transmission before symptoms. Unidentifiable features of c_b and c_d are acquired by proportional constants α and β. It is subject to the variation of above four identifiable parameters S_0, ρ, τ and ϕ that are designed to be in [0, 1]. That common base of [0, 1] allows to compare healthcare situations captured by different combinations of α and β. We leave this for future investigations of country-wise or region-wise differences. Next, the effort rate c_p (i.e. γc_b) stands for main alterations with respect to c_b. Parameter r is somewhat hired from machine replacement theory, but agreeing with a discounting effect in healthcare practice. A brief note on its literature is available in [46]. This manipulation can also be plugged into other epidemiology models similar to that of explaining COVID-19. Although parameters S_0, ρ and μ are estimated for Sri Lankan context, we convey that these are possible for initial transmission phase of many countries. Limitations arise only if required data are not reported leaving rough estimations to replace them.

We structured a computational scheme to enumerate α, β and r via the recursive relationship (16) of operating periods $(L_k; k = 1, 2, \ldots)$. As per the analysis done in Theorem 2 of [27], conditional existence and uniqueness of optimal solution given by (14) are guaranteed. However, we proceed with feasible numerical solutions. WHO time-to-time adopted country-wise risk level of COVID-19 transmission [47,48]. Thus, we are motivated to bring prevailing epidemiological situation into the model by γ and other surveillance aspects by α, β and r. Many countries have experienced mixed outcome (favorable or unfavorable) in mitigating COVID-19. Such ground-level situation can be acquired by γ. As a mechanism to that, we developed scenarios changing c_b and c_p with $\gamma > 1$ and $\gamma < 1$. One can predict the timing of ETC by different γ under these options. Rogers and Hartman [49] carried out similar experimental results for economic life time of equipment under technological change. We design γ via fuzzy membership functions, where one can easily combine more effects in interactively. We bring only two important aspects as number of PCR tests (U_a) and recovered portion (U_b). In further research, epidemiological feedback into the model can also be strengthened via an SIR model.

Statistics of unidentifiable parameters are produced with the support of bootstrap confidence intervals. We do not prefer maximum likelihoods estimates as the distributions are unknown. To make a clear platform for investigations we observe that α is more sensitive than β in deciding operating periods for a case-specific r (i.e. $r = 0.016$). Thus, changes in demand play a dominant role in practice compared to deterioration.

Continuous increase of ETC frequency (decreasing operating periods) would be an alarming situation for any healthcare system. This is because capacity may be overwhelmed by the demand for ETC. According to our observations and predictions (by mean and median), $L_1 > L_2 > L_3 > L_4 > L_5$ (ETC frequency continuously increases) and then $L_5 < L_6$ (ETC frequency started to decrease). Thus, finally some indication is given that ETC is in right direction amidst effort constraints. The tactic we used in (26) for revising r for predictions sets a baseline. One can infer more alternatives for more predictions somewhat similar to that proposed in [46]. In Sri Lanka, after the procedure ETC-4, many of the reported cases are from Navy military personnel, who initially engaged in combating operations [50]. Therefore, next ETC procedure would have been with targeted testing of Navy personnel and their close-contacts.

All investigations are based on numerical experiments starting with identifiable set of parameters, consolidating with unidentifiable parameters and then developing different scenarios. As a future work, one can carry out analytical approach whenever possible, similar to that of [51]. In that analysis, partial derivatives with respect to variables and parameters have been used to investigate cost-minimization in optimal replacement considering intensity of utilization. Such an approach would minimize possible instabilities occurred due to initial guesses provided to software tools as in the computational scheme in Section 3.3. ETC is always a concern due to the challenges made by asymptomatic cases, clinical severity and precautionary quarantine [52,53]. Therefore, any healthcare system should be equipped with informative decision making systems. We propose this work as a part of such a system, where one can customize with more specific regional attributes subject to the availability of data. Regnier et al. [54] proposed four decision methods in asset replacement namely economic life method, challenger/defender method, fixed service life method and dynamic programming. In the long-run of a disease transmission, one may analyze such different approaches in a context similar to ETC.

CRediT authorship contribution statement

R.G.U.I. Meththananda: Writing, Data handling and Simulations. **N.C. Ganegoda:** Main conceptualisation, Design of results and Initial draft. **S.S.N. Perera:** Review and Writing. **K.K.W.H. Erandi:** Writing and Simulations. **Y. Jayathunga:** Data handling and Simulation. **H.O.W. Peiris:** Data handling and Simulation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data of confirmed cases and PCR tests can be retrieved by the Epidemiology Unit (https://www.epid.gov.lk/web/) and Health Promotion Bureau of Sri Lanka (https://www.hpb.health.gov.lk/en).

Appendix. Deriving (15) from (14)

We recall Eq. (14).

$$\int_t^{x^{-1}(t)} e^{-ru}\left[e^{(c_b+c_d)t-c_du} - e^{(c_b+c_d)x(u)-c_du}\right]du = \mu e^{(c_p-r)t}. \tag{29}$$

For convenience, we abbreviate into the notation $B(t,u) = e^{(c_b+c_d)t-c_du}$. Now, by differentiating both sides with respect to t, we have;

$$\frac{d}{dt}\int_t^{x^{-1}(t)} e^{-ru}\left[B(t,u) - B(x(u),u)\right]du = \frac{d}{dt}\mu e^{(c_p-r)t}. \tag{30}$$

We need the generalized Leibnitz formula [55];

$$\frac{d}{dt}\int_{\nu(t)}^{\eta(t)} F(t,u)du = \int_{\nu(t)}^{\eta(t)} \frac{\partial}{\partial t}F(t,u)du + F(t,\eta(t))\frac{d}{dt}\eta(t) - F(t,\nu(t))\frac{d}{dt}\nu(t). \tag{31}$$

Now, we apply (31) into the left-hand side (LHS) of (30) with $F(t,u) = e^{-ru}\left[B(t,u) - B(x(u),u)\right]$, $\nu(t) = t$ and $\eta(t) = x^{-1}(t)$.

$$\text{LHS of (30)} = \int_t^{x^{-1}(t)} e^{-ru}\frac{\partial}{\partial t}B(t,u)du + e^{-rx^{-1}(t)}[B(t,x^{-1}(t))- B(x(x^{-1}(t)),x^{-1}(t))]\frac{d}{dt}x^{-1}(t) - e^{-rt}\left[B(t,t) - B(x(t),t)\right] \tag{32}$$

The second term vanishes since $x(x^{-1}(t)) = t$. Evaluation of integral term and subsequent substitutions of (32) along with determining the right-hand side of (30) yield (30) to become;

$$\frac{(c_b+c_d)}{-(r+c_d)}e^{(c_b+c_d)t}\left(e^{-(r+c_d)x^{-1}(t)} - e^{-(r+c_d)t}\right) - e^{-rt}\left(e^{c_b t} - e^{(c_b+c_d)x(t)-c_d t}\right) = \mu(c_p - r)e^{(c_p-r)t}. \tag{33}$$

Next, we carry out three step-wise multiplications to both sides: multiplying by $-(r+c_d)$, multiplying by e^{rt} and multiplying by $e^{-c_b t}$ to obtain the required formula coming in (15);

$$(r+c_d)e^{-(c_b+c_d)L(t)} - (c_b+c_d)e^{-(r+c_d)L(x^{-1}(t))} = r - c_b - \mu(r+c_d)(r-c_p)e^{(c_p-c_b)t}. \tag{34}$$

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