The Pennsylvania State University

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Department of Mathematics

An Initial Value Control Problem For The Burgers Equation

A Thesis in

Mathematics

by

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ABSTRACT

In this thesis, we consider an initial value control problem governed by the Burgers equation.

The cost functional is

$$\underset{u_c}{\mathrm{Min}}.J = \underset{u_c}{\mathrm{Min}}.\iint_{\Omega} g(x,t,u(x,t,u_c))dxdt$$

where Ω is a given domain in the upper half plane and u is the solution to the Burgers equation with initial condition

$$u(x,0) = \begin{cases} u_1 & ; \quad x < 0 \\ u_c & ; \quad 0 \leq x \leq 1 \\ u_r & ; \quad x > 1 \end{cases}$$

By imposing certain conditions on g, we establish the differentiability of J with respect to u_c and also prove a theorem providing upper and lower bounds for the controller u_c.

We then compute the distributional derivative of the cost functional J with respect to u_c, by representing the solution to the Burgers equation using Heaviside-Delta functions. We use this derivative to find the minimizing u_c.

We choose u^2 as the functional g satisfying all the assumptions and compute the cost functional and its derivative for some representative cases, by direct integration and do the same thing using the distributional derivative as well. By comparing the numerical results, we conclude the validity of our distributional approach.

Possible extensions for more general cases are also mentioned.

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