Vidyodaya J. Arts. Sci., Lett., *Vol.* 6, *No.* 2, *pp.* 72 - 75, 1978

The Longitudinal Vibrational Mode of A Tuning Fork

M. G. C. PEIRIS AND K. TENNAKONE

Department of Physics, Vidyodaya Campus

Abstract

When a sounded tuning fork is held with its lower end in contact with a solid it is erroneously assumed that the vibration transmitted to the solid has same frequency as the vibration frequency of the prongs of the fork. Experiment shows that the vibration transmitted most effectively has a frequency twice that of the prongs. A theoretical analysis is presented which shows that the prongs vibrate longitudinally, the Fourier components of this longitudinal mode has only even harmonics of the prong frequency, the most intense being the second harmonic.

When a sounded tuning fork is held near the ear, the frequency of the note heard is the frequency of vibration of the prongs. The frequency marked on a commercial tuning fork is this frequency. However, in certain experiments the lower end of the sounded tuning fork is kept in contact with some instrument and the vibration (forced or resonant) transmitted by the fork is taken as the frequency of vibration of the prongs. Sonometer experiments in the elementary physics laboratory, tuning of musical instruments and some experiments in physiological acoustics frequently utilizes the above method. We have noted that when a sounded tuning fork of frequency f (frequency of vibration of the prongs) is held with its lower end in contact with a soild object the vibration most effectively transmitted to the solid has a frequency 2f but not f. Also the other odd harmonics of the prong frequency are almost entirely absent in the transmitted vibration. This fact can be demonstrated simply by connecting a microphone to a oscilloscope. When the prongs of the vibrating tuning fork is held near the microphone the oscilloscope indicates a frequency f. However, when the lower end of the fork is held in contact with the microphone the frequency indicated is 2f. Although the presence of second harmonic in a tuning fork used in the manner we have described is noted by other workers[1] we have not found an explanation of the effect in literature.

In this work we present an extremely simple theoretical analysis which justify our observation. That is when the prongs of a tuning fork vibrates, the tuning fork as a whole undergoes longitudinal vibrations, the Fourier components of this vibration has only even harmonics of the prong frequency, the most intense being the second harmonic.

Fig. (1) indicates a displaced position of the prongs of a symmetrically vibrating tuning fork (the phase of harmonic motions of the two prongs differ by π, if the amplitudes of the prongs are same then the motion of the prongs

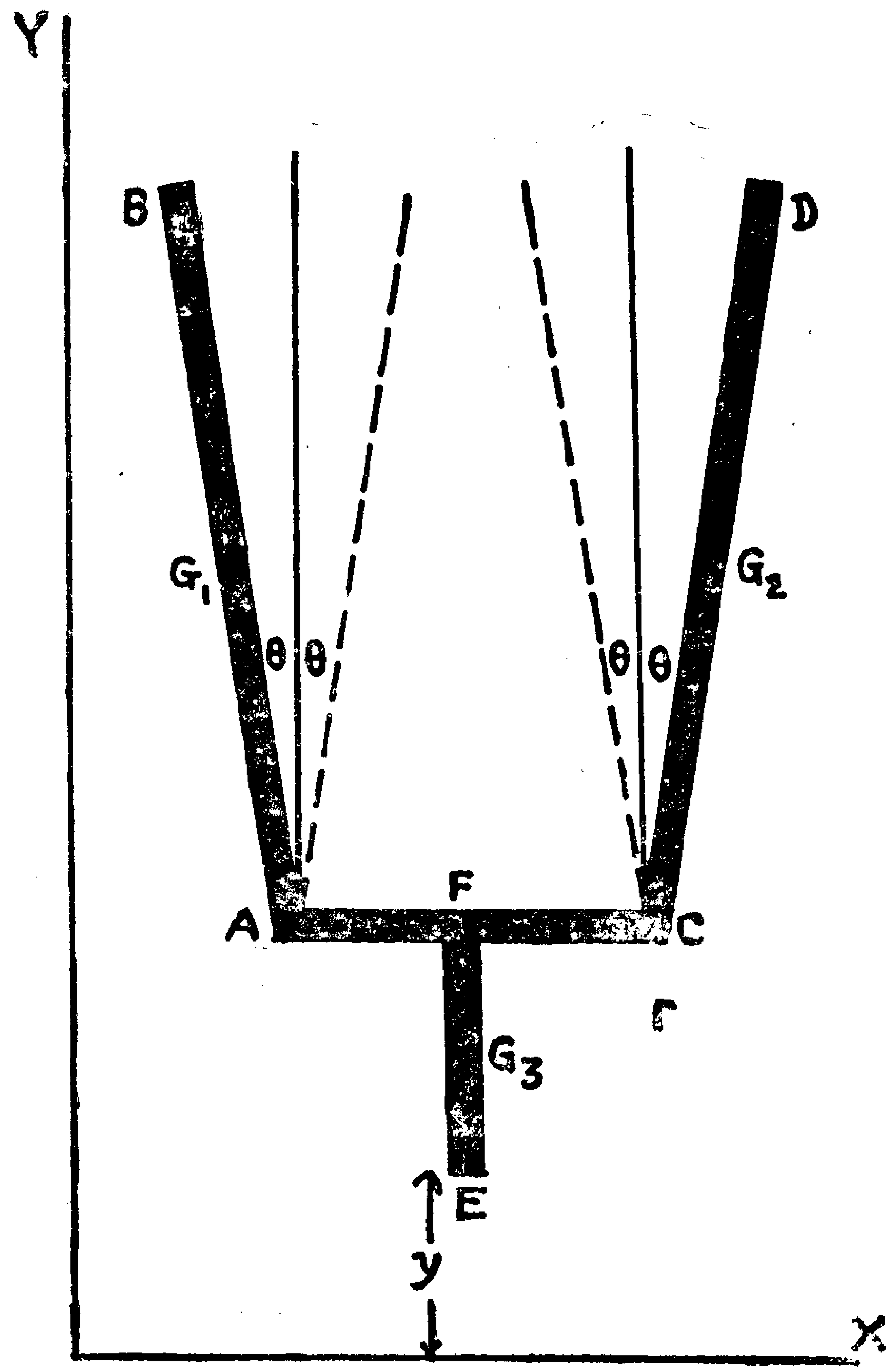

Fig. 1. Symmetrically vibrating tuning fork

are symmetrical). Let G_1, G_2 and G_3 be the centre of masses of the portions AB, CD and AFCE of the fork. Let AB=CD=2b, $FG_3=c_1$, $G_3E=c_2$, AC=2d M_1 the mass of a prong (portion AB or CD) and M_2 the mass of the portion AFCE. The frame YOX is fixed with respect to the centre of mass of the fork and let the lower end E of the fork be at a distance y above the axis OX at time t. When the prongs vibrate we have,

$$\theta = \theta_0 \sin wt, \tag{1}$$

where $w = 2\pi f$ and θ_0 = constant. The linear momentum of the fork is conserved for impulsive motion of the fork. Applying the law of conservation of linear momentum with respect to centre of mass frame in the Y direction we obtain,

$$2M_1 \frac{d}{dt} (b \cos \theta + c_1 + c_2 + y) + M_2 \frac{d}{dt} (y + c_2) = 0 \tag{2}$$

$$\text{or, } y = - \frac{2M_1 b}{2M_1 + M_2} \cos (\theta_0 \sin wt) + \text{constant.} \tag{3}$$

Using the power series expansion of cos x ($x = \theta_\circ \sin wt$) and expressing the terms of the form $\sin^{2n} wt$ in terms of cosines of the multiples of wt, we get the Fourier expansion[2] of y as,

$$y = \frac{2M_1 b}{2M_1 + M_2} \sum_{n=1}^{\infty} \left[\left\{ \sum_{m=n}^{\infty} \frac{(-1)^{m-n} \theta_\circ^{2m} \, {}^{m}C_{m-n}}{2^{2m-1} \cdot (2m)!} \right\} \cos 2nwt \right] + \text{constant} \tag{4}$$

where ${}^{m}C_{m-n} = m!/(m-n)!n!$, and n is a integer $\neq$ O. The equation (4) implies the the lower end E of the fork undergoes longitudinal vibration (or equivalently the tuning fork as a whole vibrates longitudinally) and that Fourier components of the vibration consists only of even harmonics of the prong frequency f. The amplitude θ_0 of the prong vibration cannot exceed d/2b, otherwise the prongs would touch each other. As $d/2b << 1$ (for a tuning fork commonly used in the laboratory $d/2b \simeq 0.05$), it follows that $\theta_\circ << 1$ and the expression (4) can be approximated to the form[2],

$$y = \frac{2M_1 b}{2M_1 + M_2} \sum_{n=1}^{\infty} \frac{\theta_\circ^{2n}}{2^{2n-1} (2n)!} \cos 2nwt + \text{constant} \tag{5}$$

by taking the first term ($m = n^{th}$ term) of the quantity under the double bracket in (4). Thus contribution from harmonics greater than the second is insignificant. When the amplitude of vibration of the end B or D of the fork is x ($x < d$), we have $\theta_0 = x/2b$ and the amplitude of longitudinal vibration of the end E corresponding to the second harmonic ($n = 1$) is,

$$\frac{M_1}{2M_1 + M_2} \frac{x^2}{4b}$$

Hence the amplitude of longitudinal vibrations with frequency 2f is less than that of the prong vibrations with frequency f. This accounts for the fact that when a sounded tuning fork is held near the ear the note heard is that of the vibration of the prongs. However when the sounded tuning fork is held in contact with a solid the vibration transmitted most effectively is longitudinal mode which has a frequency 2f.

In the above analysis we have assumed that the portion AFC of the turning fork remains straight when the prongs vibrate. Actually the portion AFC bulges up and down with a frequency f. The amplitude of this longitudinal vibration is less than amplitude of the mode we have discussed. However, this up and down bulging motion of the portion AFC accounts for the fact that when the turning fork is allowed rest with the end E in contact with a solid the transmitted vibration has a component with frequency f of small amplitude.

References

1. A. Wood, *Acoustics* (Blackie & Sons Limited, London 1960), p. 465.

2. The method we have used for obtaining the Fourier expension of y is simple, the conventional method need evaluation of complicated intregrals and approximations are difficult.