Vidyodaya J. Arts. Sci. Lett., *Vol.* 7, *Nos.* 1 & 2 1979, *pp.* 11–14

TEST OF EQUALITY BETWEEN SETS OF COEFFICIENTS IN TWO LINEAR REGRESSIONS AFTER A PRELIMINARY TEST OF EQUALITY OF VARIANCES

by

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1. Introduction

There is a considerable literature in Econometrics on testing equality between sets of coefficients in two linear regressions when the nature of the disturbance variances are a priori known. Chow (1960) [1] and this author (1977) [4] considered the two extreme cases of this problem. Chow suggested a test statistic to test the equality between sets of coefficients in two linear regressions assuming the disturbance variances are equal and this author suggested a test statistic to the problem assuming the disturbance variances are unequal.

In many practical situations we really do not have any idea whether the variances of the two regressions are equal or not. The purpose of the present article is to study the problem of testing equality between sets of coefficients when the nature of the disturbance variances are a priori not known by considering the problem as a combination of two statistical hypotheses.

2. Formulation of the Problem

Consider the models

$$Yi = X_i \beta_i + e_i \text{ , } e_i \backsim N(O, \sigma_i^2 I_{n_i}), \; i = 1, 2 \text{ ——— (1) and}$$

$$E e_i e_j = O \text{ for } i \neq j, \; i = j, = 1, 2,$$

Where Y_i and X_i are n_i xi and n_i x k observation matrices, βi is a kx 1 coefficient matrix, and e_i is an n_i x1 disturbance matrix for i = 1,2.

Assume all X_i' s have full column rank, ie $n_i > k$ for i = 1,2.

We wish to test the hypothesis that $\beta_1 = \beta_2$ without assuming any restrictions on σ_1^2 and σ_2^2. Thus we have

$$\Omega = \left\{ (\beta_1, \beta_2, \sigma_1^2, \sigma_2^2) \ -\infty < \beta_1, \beta_2 < \infty, 0 < \sigma_1^2, \sigma_2^2 < \infty \right\}$$

and

$$\omega = \left\{ (\beta_1, \beta_2, \sigma_1^2, \sigma_2^2), \ -\infty < \beta_1 = \beta_2, < \infty, 0 < \sigma_1^2, \sigma_2^2 < \infty \right\}$$

One possible statistic on which to base this test is the statistic which is defined by the likelihood ratio $\lambda = \dfrac{L(\hat{\omega})}{L(\hat{\Omega})}$

But difficulty arises. Unlike other likelihood ratio tests, this likelihood ratio does not define a statistic which is a function of statistic that has a well known distribution. Thus we cannot easily compute certain desirable properties, such as the significance, level of the test. The other test that can be used is the one suggested by this author in 1977 [4].

Since the test suggested in [4] is an exact test we can easily compute the significance level of the test. But if it happens that $\sigma_1^2 = \sigma_2^2$ the power of the test will be greatly reduced if this testing procedure is used. The appropriate test in this case is the Chow test. To avoid this difficulty a two step procedure can be used. In the first step, a preliminary test is performed to ascertain whether or not the variances may be regarded as equal. On the basis of the outcome of this test proceeds to test for equality of regression coefficients.

3. Result

The preliminary hypothesis that is tested is $H_{oo} : \sigma_1^2 = \sigma_2^2$. For this preliminary test the statistic

$$\frac{e_1^1 e_1 / n_1 - k}{e_2 e_2 / n_2 - k}$$ will be employed, where

e_1 and e_2 are OLS residuals from the models..........(1)

Under the null hypothesis H_{oo}

$$\frac{e_1^1 e_1 / n_1 - k}{e_2^1 e_2 / n_2 - k} \sim F_{n_2 - k}^{n_1 - k} \qquad \text{—————(2)}$$

Case 1

The hypothesis H_{oo}, is i.e. $\sigma_1^2 = \sigma_2 2$ is accepted if the computed $F \lesssim C_1$. The constant C_1 is usually selected so that, if $\sigma_1^2 = \sigma^2{}_2$.

$$\Pr \quad (F = C_1) = 1 - \alpha \quad .... (3)$$

Where α , is the desired significance level of this test. If $F \lesssim C_1$ we use Chow test to test the hypothesis H_o which is $\beta_1 = \beta_2$.

The hypothesis that $\beta_1 = \beta_2$ is rejected if the computed $F^* \gtrsim C_2$ given that Hoo is true, where the constant C_2 is selected so that

$$\alpha_2 = \Pr (F^* \gtrsim C_2) \quad .... (4)$$

is the desired significance level of the test and where F^* is the F distribution obtain from the Chow test. If computed $F^* \lesssim C_2$ given that H_{oo} is true the hypothesis $\beta_1 = \beta_2$ is accepted.

If we use this two steps procedure for testing the hypothesis the question that naturally arises is this :

Does $\alpha 2$ in (4) represent the correct significance level for the second test because the test depends on the outcome of the preliminary test. It happens that this is very easy to answer because we can show that, if $\sigma_1^2 = \sigma_2^2$ the two statistics F and F^* are stochastically independent and hence α 2 represents the correct significance level for the test (4).

Case 2

The hypothesis H_{oo}, i.e. $\sigma_1^2 = \sigma_2^2$ is rejected if the computed $F \gtrsim C_1$ where C_1 is defined as in (3).. In this case we can use the test suggested by the author in 1977. Let F^{**} be the F statistic of this test.

The hypothesis that $\beta_1 = \beta_2$ is rejected if the computed $F^{**} \gtrsim C_3$, where the constant C_3 is selected so that

$$\alpha_3 = \Pr (F^{**} \gtrsim C_3) \quad .... (5)$$

is the desired significance level of the test. If computed $F^{**} \lesssim C_3$ the hypothesis $\beta_1 = \beta_2$ is accepted.

But in this case these two statistics F and F^{**} may be dependent, If so α_3 does not represent the correct significance level for the test (5).

References

1. Chow, G. C., "Tests of Equality Between Sets of Coefficients in Two Linear Regressions", *Econometrica,* 28, (1960), 591-605.

2. Gurland, J. and McCullough, R. S., 'Testing Equality of Means After a Preliminary Tests of Equality of Variances", *Biometrica,* 49, (1962), 403 - 417.

3. Hogg, R. V. and Craig, A. T., "*Introduction to Mathematical Statistics,* New York, The Macmillan Co., 1965.

4. Jayatissa, W. A., "Tests of Equality Between Sets of Coefficients in Two Linear Regressions when the Disturbance Variances, are Unequal", *Econometrica,* Vol. 45, No. 5 (July 1977), 1291 - 1292.