A Modified Firefly Algorithm to solve Univariate Nonlinear Equations with Complex Roots

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Abstract— **Recently developed meta-heuristic algorithms such as firefly algorithm, bat algorithm, particle swarm optimization and harmony search are now becoming popular for providing nearly accurate solutions for tough optimization problems. This paper addresses the problem of finding all roots of a given univariate nonlinear equation with real and complex roots using a modified firefly algorithm (MOD FA). The appropriate modifications are applied to the existing firefly algorithm (FA) by introducing an archive. Better fireflies are noted and stored in the archive during the iteration process and then their positions are replaced by new random ones. A comparison was carried out with the original firefly algorithm and also with the genetic algorithm (GA) which has a similar behaviour to the firefly algorithm. Computer simulations show that the proposed firefly algorithm performs well in solving nonlinear equations with real and complex roots within a specified region. The suggested method can be further extended to solve a given system of nonlinear equations.**

Keywords— **Firefly Algorithm; Nonlinear Equations; Archive; Real Roots; Complex Roots.**

I. INTRODUCTION

Solution of a single variable nonlinear equation can be defined as finding x, where $f(x) = 0$. Problems requiring the solutions of such nonlinear equations arise frequently in the fields of mathematics, engineering, physics and computer science. When thinking of a single nonlinear equation, solving $f(x) = 0$ is not easy, though it can be done in simple cases like finding roots of quadratic equations. If the function is complicated, approximations can be made using iterative procedures which are also known as numerical methods. Having their own drawbacks, none of these numerical approaches appear to be able to solve all types of nonlinear equations, especially when it has more than one solution. When the equation has complex solutions the situation is even worse. Most of the existing numerical approaches are associated with derivative of the function and thus solving equations with non-differentiable nonlinear functions like "Weierstrass function" is a challenge when the roots are needed in a specified region [1]. On the other hand, approaches like bisection method do not require derivative information but cannot be used in finding approximations although the given function is continuous on [a, b] but when not having opposite signs for $f(a)$ and $f(b)$ [2]. The Newton's method needs derivative information but fails in finding roots when multiple roots or very close roots exist in an equation [2]. The other main disadvantage of these numerical approaches is their inability to find more than one solution at a time, in a specified region. As such, it is clear that finding better algorithms to determine more than one root or all available roots of a function, simultaneously, without having to use its derivative information has become the need of the hour.

The remarkable performance of nature inspired algorithms over other classical optimization techniques encourages researchers to apply them for various difficult optimization problems. Recently developed algorithms like firefly algorithm, bat algorithm, cuckoo search and artificial bees' colony have proved their success over many difficult problems [3, 4, 5, 6, 13].

Firefly algorithm (FA) is one of the nature-inspired meta-heuristic algorithms developed by Xin She Yang, originally designed to solve continuous optimization problems [7]. It is capable of giving several possible approximations for a given problem rather than giving one globally best solution as in Bat Algorithm or Particle Swarm Optimization (PSO) algorithm. Since our problem of interest is also associated with getting more than one optimal solution in a specified region, we have selected the firefly algorithm. Our problem of interest can be defined as follows.

Let f be a function $s.t. f: D \to R$ where $D \subset C$. Neither the differentiability nor the continuity of f is required. The problem is to find all $x \in D\ s.t. f(x) = 0$. Since the problem is handled as an optimization problem, the problem becomes finding $x \in D\ s.t. |f(x)| = 0$. However it should be emphasized here that, since the function $f(x)$ may have multiple roots, the optimization problem $|f(x)| = 0$, also will have multiple optimal solutions. Therefore our objective turns out to be finding all such optimal solutions.

In this research, modifications to the original firefly algorithm are introduced to solve nonlinear equations with more than one real and/or complex root within a specified region. The modifications have been done by applying a similar concept to the elitism in Genetic Algorithms (GAs). In an iteration, the better fireflies who serve as solutions are selected and they are put into an archive while replacing their positions with random fireflies. The iterative process of the original firefly algorithm is also changed using a "flag." Single variable nonlinear equations that contain real and/or complex roots within a specified region were tested with the algorithm to make conclusions.