

**Interpretation of Magnetic anomalies using Backus-Gilbert
Inversion Technique**

by

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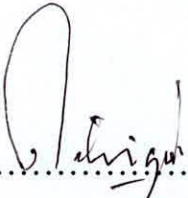
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ABSTRACT

Numerical investigations have been carried out to formulate a suitable modeling scheme for magnetic anomalies in terms of sub-surface magnetized structures, based upon Backus-Gilbert inversion method. Magnetic anomalies due to two dimensional subsurface structures of known size and shape have been computed using a well-established forward modeling method. Each anomaly calculated was added a certain percentage of noise so that artificially generated data resemble real data. Then the Backus-Gilbert method suitably formulated for the present purpose was used to model these magnetic anomalies. Results of the modeling were compared with the dimensions of the causative bodies used for the forward modeling and found that the modeling method works satisfactorily. Even though the method presented in this thesis restricted to two-dimensional bodies, it can be extended to three dimensional bodies with some modifications.

The proposed method involves inversion of large matrices and most of the time these matrices are ill conditioned. In such cases singular value decomposition was used to avoid such problems and improve the results.

Numerical experiments have been carried out successfully to explore the possibility of modeling magnetic anomalies using the Backus-Gilbert method in terms of

(1) Igneous intrusions and magnetized structures resembling sedimentary basins in terms of bodies with a constant magnetization,

(2) Igneous intrusions and magnetized structures resembling sedimentary basins with magnetizations decrease with depth according to a relationship close to an assumed relationship,

(3) Igneous intrusions and magnetized structures resembling sedimentary basins consist of layers having different magnetizations (constant within each layer, but vary among layers) by constructing averaging kernels, and

(4) Magnetized geological fault bodies consisting of horizontal layers with different magnetization by constructing averaging kernels.

This method may have useful applications in the oil and mineral industry where modeling of magnetic and gravity anomalies play an important role in looking for structures favorable for accumulation of oil or minerals.

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