

# Interpretation of Potential field data using Backus and Gilbert Inversion Technique

by

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“The work described in this thesis was carried out by me under the supervision of Professor D.A.Tantrigoda and Professor Arjuna de Zoysa and a report on this has not been submitted in whole or in part to any University or any other institution for another Degree/Diploma”.


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## Table of Contents

Table of Contents .....	i
List of Figures .....	iv
List of Tables.....	ix
List of Appendices .....	x
Acknowledgements.....	xi
Abstract .....	xii
CHAPTER 1 .....	1
Introduction.....	1
1.1. Introduction .....	1
1.2. Gravitation.....	2
1.3. Gravity Modeling .....	3
1.4. Description of Some Important Terminology .....	4
1.4.1. Forward problem.....	4
1.4.2. Inverse problem.....	5
1.5. Objectives of the study .....	5
1.6. Structure of the thesis .....	6
CHAPTER 2 .....	8
Literature Review.....	8
2.1. Introduction .....	8
2.2. Linear Inverse problems: The discrete case .....	10
2.3. Existence, uniqueness and stability of the inverse solution .....	11
2.4. Types of inverse problems .....	13
2.4.1. Even determined system .....	13
2.4.2. Over determined system.....	13

2.4.3.	Under determined system.....	17
2.4.4.	Mixed determined system .....	19
2.4.5.	Singular value decomposition.....	21
2.4.5.1.	Properties of the generalized inverse.....	23
2.5.	The data resolution and model resolution matrices.....	25
CHAPTER 3 .....		27
Backus and Gilbert Method .....		27
3.1.	Introduction .....	27
3.1.1.	Introduction of derivation of the Backus and Gilbert method .....	30
3.1.2.	Application of the Backus and Gilbert technique for gravity inversions..	32
3.2.	Averaging Kernel .....	36
3.2.1.	Detailed derivation of Averaging Kernel.....	38
CHAPTER 4 .....		42
Application of Backus and Gilbert Method for Gravity Interpretation.....		42
4.1.	Linear Gravity Models .....	42
4.2.	Modeling of bodies with single density .....	44
4.3.	Modeling of bodies with density vary with depth closest to a given relationship .....	45
4.4.	Modeling of gravity anomalies in terms of bodies having density vary with depth in terms of local averages using constructing averaging kernels .....	47
CHAPTER 5 .....		49
Investigation of possibility of modeling Gravity Anomalies in terms of bodies with density varies with depth using the Backus and Gilbert Method.....		49
5.1.	Introduction .....	49
5.2.	Modeling of bodies with a single known density .....	50
5.2.1.	Modeling of gravity anomalies due to sedimentary basins in terms of bodies with constant density .....	51

5.2.2. Modeling of gravity anomalies due to igneous intrusions in terms of bodies with constant density .....	58
5.3. Modeling of bodies with density contrast varying with depth according to a known linear function.....	64
5.3.1. Modeling of gravity anomalies caused by sedimentary basins with densities increases with depth.....	66
5.3.2. Modeling of gravity anomalies caused by igneous intrusions with densities increases with depth.....	74
5.4. Modeling of gravity anomalies in terms of bodies (sedimentary basins) with density contrast varies with depth in terms of local averages constructing averaging kernels .....	82
5.5. Modeling of gravity anomalies in terms of bodies (igneous intrusions) with density contrast varies with depth in terms of local averages constructing averaging kernels .....	87
5.6. Modeling of gravity anomalies over a sedimentary basin having a horizontal igneous layer intruded into it.....	91
5.7. Modeling of vertical density distribution in a normal geological fault.....	99
5.8. Modeling of Real Data .....	104
 CHAPTER 6 .....	 109
Conclusions and Discussion.....	109
6.1 Introduction .....	109
6.2 Conclusions .....	109
 REFERENCES.....	 112
 APPENDICES .....	 115



## List of Figures

Figure 2-1: Connecting link between a data and the corresponding model space in a forward and an inverse problem.....	8
Figure 2-2: The graphical representation of an estimation, an appraisal and forward problem .....	9
Figure 2-3: Illustration of Least Squares method using vector spaces.....	16
Figure 2-4: Illustration of $U_p$ , $V_p$ , $U_0$ and $V_0$ .....	24
Figure 4-1: Linear model for inversion of gravity anomalies .....	43
Figure 4-2: Linear model for inversion of gravity anomalies .....	44
Figure 4-3: Division of a body to calculate its gravity anomaly.....	44
Figure 4-4: Graphical representation of assumed body .....	46
Figure 4-5: The model consisting of 25 strips .....	48
Figure 5-1: The shape of the body (collection of cells in green) used for generation of synthetic data which were later used as observed data. ....	52
Figure 5-2: The shape of the initial trial body of the iteration process.....	52
Figure 5-3: Gravity model obtained after the first iteration and the gravity anomaly it produces (blue dashed line).....	53
Figure 5-4: Gravity model obtained after the second iteration and the gravity anomaly it produces (blue dashed line).....	53
Figure 5-5: Gravity model obtained after the third iteration and the gravity anomaly it produces (blue dashed line).....	54
Figure 5-6: Gravity model obtained after the fourth iteration and the gravity anomaly it produces (blue dashed line).....	54
Figure 5-7: Gravity model obtained after the fifth iteration and the gravity anomaly it produces (blue dashed line).....	55
Figure 5-8: The observed gravity anomaly with 3% error .....	56
Figure 5-9 Gravity model obtained after the first iteration and the gravity anomaly it produces (blue dashed line).....	56
Figure 5-10: Gravity model obtained after the third iteration and the gravity anomaly it produces (blue dashed line).....	57

Figure 5-11: Gravity model obtained after the fifth iteration and the gravity anomaly it produces (blue dashed line).....	57
Figure 5-12: The shape of the body (collection of cells in green) used for generation of synthetic data which were later used as observed data. ....	59
Figure 5-13: The shape of the initial trial body of the iteration process. ....	59
Figure 5-14: Gravity model obtained after the first iteration and the gravity anomaly it produces (blue dashed line).....	60
Figure 5-15: Gravity model obtained after the second iteration and the gravity anomaly it produces (blue dashed line). ....	60
Figure 5-16: Gravity model obtained after the third iteration and the gravity anomaly it produces (blue dashed line).....	61
Figure 5-17: Gravity model obtained after the fourth iteration and the gravity anomaly it produces (blue dashed line).....	61
Figure 5-18: The observed gravity anomaly with 2% error. ....	62
Figure 5-19: Gravity model obtained after the first iteration and the gravity anomaly it produces (blue dashed line).....	63
Figure 5-20: Gravity model obtained after the third iteration and the gravity anomaly it produces (blue dashed line).....	63
Figure 5-21: Gravity model obtained after the sixth iteration and the gravity anomaly it produces (blue dashed line).....	64
Figure 5-22: The shape of the body (collection of 5 strips) used for generation of synthetic data which were later used as observed data. ....	67
Figure 5-23: The shape of the initial trial body of the iteration process. ....	68
Figure 5-24: Gravity model obtained after the first iteration and the gravity anomaly it produces (blue dashed line).....	68
Figure 5-25: Gravity model obtained after the second iteration and the gravity anomaly it produces (blue dashed line). ....	69
Figure 5-26: Gravity model obtained after the third iteration and the gravity anomaly it produces (blue dashed line).....	69
Figure 5-27: Gravity model obtained after the fourth iteration and the gravity anomaly it produces (blue dashed line).....	70

Figure 5-28: Gravity model obtained after the fifth iteration and the gravity anomaly it produces (blue dashed line).....	70
Figure 5-29: The model divided into 25 strips.....	71
Figure 5-30: The density contrast variation between the assumed 5 layers (blue line) and calculated 25 layers (red line). ....	71
Figure 5-31: The observed gravity anomaly with 2% error.....	72
Figure 5-32: Gravity model obtained after the third iteration and the gravity anomaly it produces (blue dashed line).....	73
Figure 5-33: Gravity model obtained after the fifth iteration and the gravity anomaly it produces (blue dashed line).....	73
Figure 5-34: The density contrast variation between the assumed 5 layers (blue line) and calculated 25 layers after adding error (red line). ....	74
Figure 5-35: The shape of the body (collection of 5 strips) used for generation of synthetic data which were later used as observed data. ....	76
Figure 5-36: The shape of the initial trial body of the iteration process. ....	76
Figure 5-37: Gravity model obtained after the first iteration and the gravity anomaly it produces (blue dashed line).....	77
Figure 5-38: Gravity model obtained after the second iteration and the gravity anomaly it produces (blue dashed line). ....	77
Figure 5-39: Gravity model obtained after the third iteration and the gravity anomaly it produces (blue dashed line).....	78
Figure 5-40: Gravity model obtained after the fourth iteration and the gravity anomaly it produces (blue dashed line).....	78
Figure 5-41: The model divided into 25 strips.....	79
Figure 5-42: The density contrast variation between the assumed 5 layers (blue line) and calculated 25 layers (red line). ....	79
Figure 5-43: The observed generated gravity anomaly with 2% error .....	80
Figure 5-44: Gravity model obtained after the second iteration and the gravity anomaly it produces (blue dashed line). ....	81
Figure 5-45: Gravity model obtained after the fifth iteration and the gravity anomaly it produces (blue dashed line).....	81

Figure 5-46: The density contrast variation between the assumed 5 layers (blue line) and calculated 25 layers after adding error (red line) .....	82
Figure 5-47: The shape of the body (collection of 5 strips) used for generation of synthetic data which were later used as observed data. ....	84
Figure 5-48: Gravity model obtained after the fifth iteration and the gravity anomaly it produces (blue dashed line).....	84
Figure 5-49: The density contrast variation between the assumed 25 layers (blue line) and calculated 25 layers (red line). ....	85
Figure 5-50: The density contrast variation between the assumed 5 layers (blue line) and calculated 25 layers after adding error (red line). ....	87
Figure 5-51: The shape of the body (collection of 5 strips) used for generation of synthetic data which were later used as observed data. ....	88
Figure 5-52: Gravity model obtained after the fifth iteration and the gravity anomaly it produces (blue dashed line).....	88
Figure 5-53: The density contrast variation between the assumed 5 layers (blue line) and calculated 25 layers (red line). ....	89
Figure 5-54: The density contrast variation between the assumed 5 layers (blue line) and calculated 25 layers after adding 2% error (red line). ....	91
Figure 5-55: The shape of the body (collection of 5 strips) used for generation of synthetic data which were later used as observed data. ....	93
Figure 5-56: Gravity model obtained after the first iteration and the gravity anomaly it produces (blue dashed line).....	95
Figure 5-57: Gravity model obtained after the fourth iteration and the gravity anomaly it produces (blue dashed line).....	95
Figure 5-58: Gravity model obtained after the sixth iteration and the gravity anomaly it produces (blue dashed line).....	96
Figure 5-59: The model divided into 22 strips.....	96
Figure 5-60: The density contrast variation between the assumed 5 layers (blue line) and calculated 22 layers (red line) .....	97
Figure 5-61: The model divided into 30 strips.....	97

Figure 5-62: The density contrast variation between the assumed 5 layers (blue line) and calculated 25 layers (red line) .....	98
Figure 5-63: The model divided into 30 strips.....	98
Figure 5-64: The density contrast variation between the assumed 5 layers (blue line) and calculated 30 layers (red line) .....	99
Figure 5-65: The shape of the body (collection of 2D semi-infinite slabs) and the gravity anomaly it produces. ....	101
Figure 5-66: The shape of the initial trial body of the iteration process. ....	101
Figure 5-67: Gravity model obtained after the first iteration and the gravity anomaly it produces (blue dashed line).....	102
Figure 5-68: Gravity model obtained after the fifth iteration and the gravity anomaly it produces (blue dashed line).....	102
Figure 5-69: The model divided into 25 strips.....	103
Figure 5-70: The density contrast variation between the assumed 5 layers (blue line) and calculated 25 layers (red line) .....	103
Figure 5-71: Derived observed gravity anomaly from $c - c'$ cross section.....	104
Figure 5-72: The shape of the initial trial body of the iteration process. ....	105
Figure 5-73: Gravity model obtained after the first iteration and the gravity anomaly it produces (blue dashed line).....	105
Figure 5-74: Gravity model obtained after the second iteration and the gravity anomaly it produces (blue dashed line). ....	106
Figure 5-75: Gravity model obtained after the third iteration and the gravity anomaly it produces (blue dashed line).....	106
Figure 5-76: Gravity model obtained after the fifth iteration and the gravity anomaly it produces (blue dashed line).....	107
Figure 5-77: Comparison between (a) computed model and (b) model produced by Tantrigoda and Geekiyanage (1991).....	107
Figure 5-78: The model divided into 24 strips.....	108
Figure 5-79: The density contrast variation between the assumed 24 layers (blue line) and calculated 24 layers (red line). ....	108

## List of Tables

Table 5-1: The calculated densities of 25 strips.....	71
Table 5-2: The calculated densities of 25 strips after adding 2% error. ....	74
Table 5-3: The calculated densities of 25 strips.....	79
Table 5-4: The calculated densities of 25 strips after adding 2% error .....	82
Table 5-5: The assumed and calculated density values of each strip.....	85
Table 5-6: The assumed and calculated density values of each strip with 2% error .....	86
Table 5-7: The assumed and calculated density values of 25 strips .....	89
Table 5-8: The assumed and calculated densities of each strip with 2% error .....	90
Table 5-9: The calculated densities of each strip .....	97
Table 5-10: The calculated densities of 30 strips.....	98
Table 5-11: The calculated densities of each strip.....	99
Table 5-12: The calculated density values of each semi infinite slab.....	103
Table 5-13: The calculated densities of 24 strips.....	108

## List of Appendices

Appendix 1 - Derivation of the expressions.....	115
Appendix 2 - Functional spaces of geophysical data and model .....	124
Appendix 3 - Derivation of averaging kernel .....	131
Appendix 4 - Averaging kernels .....	136
Appendix 5 - Computer program for construction of single density bodies.....	145
Appendix 6 - Computer program for construction of bodies with density increases with depth using averaging kernels.....	153
Appendix 7 - Computer program for construction of bodies (geological fault) with density increases with depth .....	155
Appendix 8 - Computer program for construction of averaging kernels for bodies (geological fault) .....	157

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## **Technique**

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### **ABSTRACT**

The main objective of this study is to test the hypothesis that gravity anomalies could be modeled in terms of bodies having density that vary with depth using the Backus and Gilbert inversion method. The hypothesis was tested using a series of numerical experiments involving calculation of the gravity anomaly due to model geological structures of known dimensions and known vertical density distributions and then modeling them using the Backus and Gilbert method. This in fact is a problem in the linear inverse theory and such problems are ill posed in general. To overcome this difficulty the Singular Value Decomposition method has been successfully adopted.

The study was conducted with simulated data and to make them resemble real observations, a certain amount of random errors were added. To recover the assumed structure of the causative body, "weighted distance" minimization was applied at the latter stages of the inversion process. The method presented in this study can be used successfully to model gravity anomalies caused by;

- (1) Common geological structures (sedimentary basins, igneous intrusions and geological faults) with densities that vary with depth closest to a given relationship and

(2) A sedimentary basin of constant density having a high dense igneous layer intruded into it. Density variations with depth have been expressed in terms of local averages using averaging kernels.

This method may have useful applications in the oil and gas exploration work especially in view of its ability to identify thin igneous layers intruded in to sedimentary basins.