A MATHEMATICAL MODEL
FOR
POPULATION AGING

by

MUTHU RAMARAJA SRISKANDARAJA

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The work described in this thesis was carried out by me under the supervision of

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and a report on this has not been submitted in whole or in part to any other institution for another Degree or Diploma

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ABSTRACT

A partial differential equation was constructed and modeled by considering the flow of contaminant in an ideal lake and a sequence of valid arguments. By introducing population terms and considering an individual as a particle of contaminant in the life's river, we introduce partial differential equation whose solution gives the amount of age Z. We restricted our developed partial differential equation to the female population by introducing fertility function, sex ratio and fertile periods. If the female population is known, then using the sex ratio, the total population could be calculated and the solution for the derived partial differential equation was obtained. This solution contains fertility function, probability of survival and the instantaneous growth rate.

Using the solution and a sequence of valid arguments it can be shown that a particular condition should be satisfied by the growth rate. The value of the growth rate was found by using the fertility function, probability of survival from the data set, and an approximation. The growth rate found from the data set and the values of the growth rate were compared. It was observed that 04 % relative error in the growth rate.

CHAPTER 01

INTRODUCTION

1.1 Who is Old ?

When does someone become "old" or "elderly"? A number of terms are used to describe people considered old, but there is an increasing awareness that the terms used should acknowledge the tremendous diversity inherent in a group of people whose ages can span a range of 40 or more years.

Usually, "older people" and "older population" refer to people aged 65 or older; the "oldest old" refers to people aged 80 or older.

1.2 The Global Process of Aging

The world's older population has been growing more numerous for centuries, but the pace of growth has accelerated. The global population aged 65 or older was estimated at 461 million in 2004, an increase of 10.3 million just since 2003." Projections suggest that the annual net gain will continue to exceed 10 million over the next decade - more than 850,000 each month.

There are several demographic indices of aging - the aging index, median age, and support ratios - that compare different portions of a given population. One straightforward indicator of age structure is the aging index, defined as the number of people aged 65 or older per 100 children under age 15.

1.3 The Oldest Old

The older population within countries is also aging. Over time, a nation's older population often grows older on average as larger proportions survive to advanced ages. The "oldest old" (people with age 80 or older) constituted 18 percent of the world's older people in 2004. In many countries, the oldest old are the fastest growing segment of the population.

1.4 The Demographic Drivers of Aging.

"Why do populations age?" most people intuitively think of changes in longevity. We know that life expectancy has been rising in most countries throughout the world, so it seems reasonable that population aging is an out come of people living longer. Yet, the most prominent historical factor in population aging has been declining fertility. When thinking of population aging as an increase in the percent of people age 65 or older, it can be realized that, over time, a decline in the number of babies will mean fewer young people and proportionally more people at older ages.

1.5 Fertility

The decrease in fertility in industrialized nations during the last century has pushed the average number of children per woman in almost all more developed countries below the population replacement level of 2.1 children per women [17].

1.6 Importance of an Age Structured Population Model

The importance of age structure is clearly understood and therefore in planning for its future, a nation needs to be known, among other things, how many schools to be built, how many hospitals to be built, how many teachers and doctors to be trained, and how much taxes to be collected for social security and medical care. All these things require knowledge of the population's age structure - how many old people there will be, how many children there will be, the size of the potential labour force. Whether the logistic model of population growth is true or false, the best it can do is to predict a total population at time t . It cannot predict the number of people aged z within that population. Therefore building a new model, that is capable of providing such information become a necessity. So the target is to develop an Age Structured Population Model which would be a powerful tool for policy makers world wide. The Age Structure Model is developed considering life as a fast flowing river.