

**FITTING MATHEMATICAL MODELS FOR  
RAINFALL-RUNOFF RELATIONSHIPS**

**BY**

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## DECLARATION

I declare that this research project was carried out by me at the University of Sri Jayewardenepura under the supervision of Mr. P. Dias, Dr. G. K. Watugala, Dr. L. M. Liyanage and the report on this has not been submitted to any University for another Degree/Diploma.



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W. G. Samanthi

## Abstract

The determination of relationship between the rainfall over a catchment area and the resulting runoff is a fundamental problem for the hydrologists. Rainfall-Runoff relationships can be used to understand the catchment yields. In addition, long term records of runoff are needed for planning, designing and operation of reservoirs, river basin projects, irrigation, hydropower generation, flood control etc. Usually, there are plenty of rainfall records, but more elaborate and expensive runoff records are often limited and rarely available for a specific river under investigation in Sri Lanka as well as in the other countries. Therefore, estimation of the runoff from the observed rainfall is one of the primary challenges faced by hydrologists all over the world.

Here we study the problem of fitting mathematical models for Rainfall-Runoff Relationships for estimating runoff using available rainfall data. Three large river catchments were selected for this study. They are Kelani Ganga Basin at Hanwella gauging station, Menik Ganga Basin at Kataragama gauging station, and Gin Ganga basin at Tawalama catchment. Rainfall data are collected from the Meteorology Department of Sri Lanka, and corresponding runoff data are collected from the Irrigation Department of Sri Lanka. This study is primarily done using monthly data and it is extended by using daily data. Period of data collection was limited to 10 years-15 years.

There is a strong positive correlation between monthly rainfall and runoff; but any significant correlation between daily rainfall and runoff could not be seen.

By multiple regression, the relationships between rainfall and runoff were derived for Kelani Ganga Basin at Hanwella catchment area, Menik Ganga Basin at Kataraga catchment area and Gin Ganga Basin at Tawalama catchment area. The three models fitted here have higher efficiencies around 70% in both calibration and verification. Hence these models are suitable for the estimation of runoff using observed rainfall data.

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# *Chapter 1*

## **INTRODUCTION**

## ***1.1 The Hydrological Cycle***

The hydrological cycle is the descriptive term applied to the general circulation of water from the seas to the atmosphere, to the ground and back to the seas again. The driving force of the circulation is derived from the radiant energy received from the Sun. Fig.1.0 shows the hydrological cycle.

The cycle may be considered to begin with the water of the oceans. Water from the ocean surface is evaporated into the atmosphere. This vapour is condensed by various processes and falls to the earth as precipitation. Some of this precipitation falls directly on the seas and some falls on land surfaces. A portion of that falling on the land is retained temporarily in the soil, in surface depressions, and on vegetation and other objects until it is returned to the atmosphere by evaporation and transpiration. The remainder, moving by devious surface and underground channels to rivers, lakes and eventually to the sea, is likewise subject to evaporation and transpiration throughout its travel.

Actually, all phases of the cycle are occurring simultaneously. On world –wide basis the volumes of moisture involved in each phase of the cycle are relatively constant; but viewed in terms of a limited area, such as a small river basin, the quantities in any part of the cycle vary through wide limits. For example, a temporary unbalance of the cycle in which great volumes of water are concentrated stream result a flood. Conversely, the

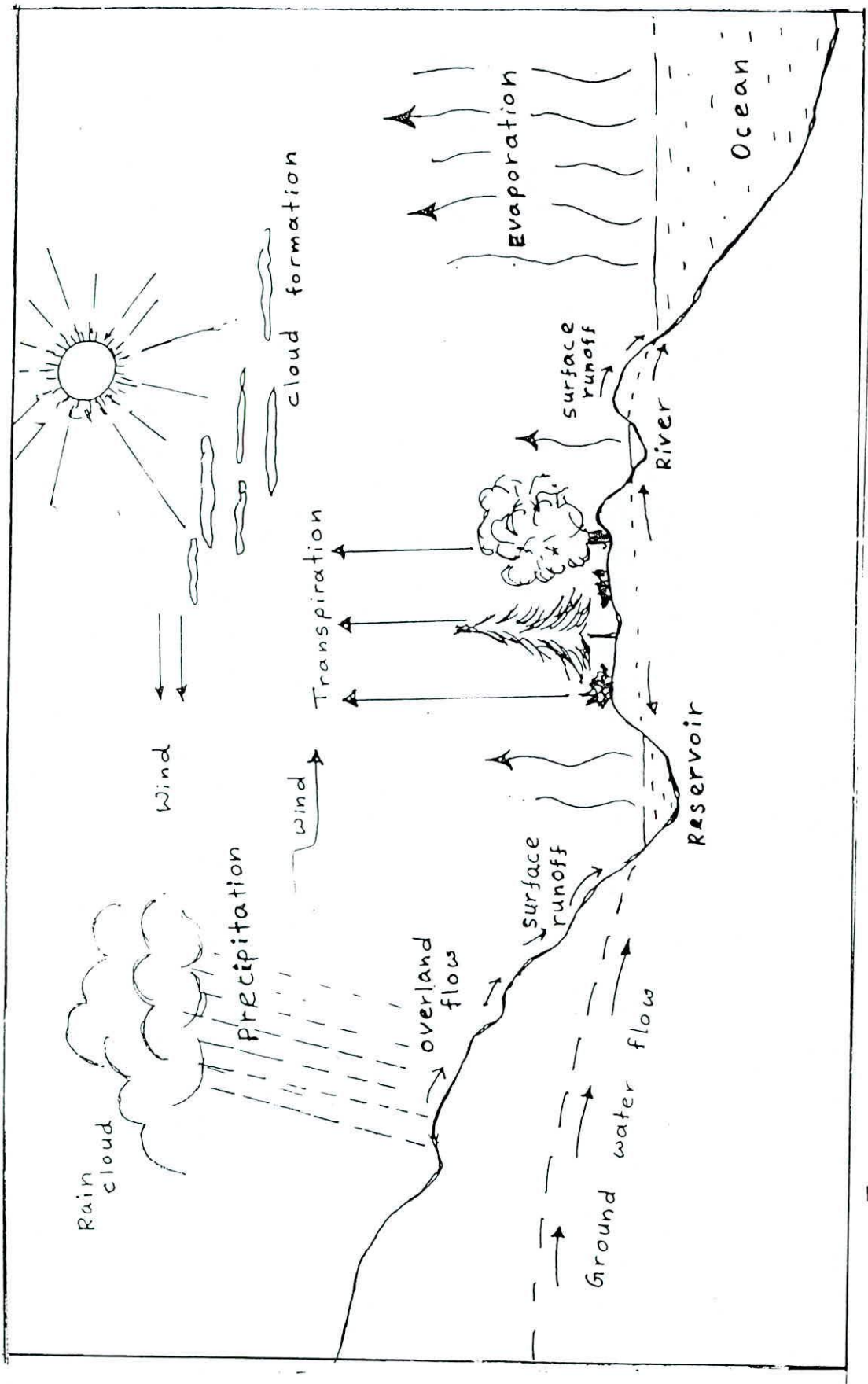


Fig. 1.0 The Hydrological Cycle

small or negligible amounts of water in the precipitation phase of the cycle lead to drought [6].

Life on Earth's surface is greatly affected by that type of variations. Hence an attempt should be made to analyse them. First step of analysing them is studying about rainfall and runoff.

## ***1.2 Rainfall and Runoff***

Water is greatly needed for the existence of life on earth surface; but it is flowing uncontrolled over land. Therefore, the available supply cannot meet the needs of the increasing population. Hence, man has to find ways of utilizing every drop of water before it returns to the sea.

The route followed by a water particle from the time it reaches the ground until it enters a stream channel is devious. It is convenient to visualize three main routes of travel;

Overland flow (surface runoff)

Inter flow

Ground water flow

Overland flow or runoff or discharge is that water which travels over the ground surface to a channel. The word channel as used here refers to any depression, which



may carry a small rivulet of water in turbulent flow during a rain, and for a short while after.

At the beginning of the rainfall, the river level and hence the runoff is low and a period of time elapses before the river begins to rise. During this period the rainfall is being intercepted by vegetation or in soaking into the ground and making up soil-moisture deficits. When the rainfall had made up catchment deficits and when surface and soil are saturated, the rain begins to contribute to the runoff. However, the runoff depends very much on the geological structure and composition of the catchment.

The derivation of relationship between the rainfall over a catchment area and the resulting runoff is a fundamental problem for the hydrologist. Rainfall – Runoff Relationships can be used to understand the catchment yields.

### ***1.3 Hydrological Modelling***

Estimation of the runoff from the observed rainfall is one of the primary challenges faced by hydrologists in the world. In terms of modern terminology, this procedure is known as hydrological modelling.

Hydrological modelling is an important tool for estimating and organizing quantitative information for the water resources planning, design and operation. Monthly runoff is required for planning of river basin projects. Long term records of