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MODELING SINHALA CHARACTERS USING CUBIC RATIONAL BÉZIER CURVES

A Thesis

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ABSTRACT

Sinhala characters have many sections that are nearly circular or elliptical. Because of this peculiarity they are good candidates for modeling by cubic rational Bézier curves. Bézier curves are drawn by taking weighted averages of coordinates of four control points.

In this study, we have modeled several Sinhala characters using a series of cubic rational Bézier curves. In each case, the character was broken down to sections depending on the change in curvature.

Access database was created which consisted of control points and their weights of sections of characters. This database is used for the implementation of Visual Basic program. This software displays the character when the user requests it by selecting the character from a menu.

Generated Sinhala charcters are very close to the Sinhala characters, which are used in these days. Therefore we can conclude that Sinhala characters can be created by using composite cubic rational Bézier curves.

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Chapter 1

INTRODUCTION

Collection of characters can be described as a Font. The differences between fonts depend mainly on the shape and the size of characters. Today there are so many fonts available for typesetting purposes and different methods have been used to create characters. Times New Roman, Arial and Courier are the well-known English fonts and FS Anuradha, Thibus, Sarasavi and Araliya are the popular Sinhala fonts.

Sinhala characters have many sections that are nearly circular or elliptical. Because of this they are good candidates for modeling by rational Bézier curves. It should be noted that nonrational Bézier curves cannot be used to represent conic sections such as parabolas, ellipse or circles exactly.

Bézier curves and surfaces are attributed and named after a French engineer named Pierre Bézier, who used them for developing a software system called 'UNISURF' in 1962. He used it for the design of the body of the Renault car.

Bézier curves have since obtained dominance in the typesetting industry and in particular with the Adobe Postscript and other font products. Bézier curves were also independently developed by P. deCastelijau of the French Car Company Citroen (about 1959), which used it as a part of its CAD system. The Bézier UNISURF system was soon published in the literature and as a result the curves now bear Bézier's name.

Cubic splines are based on interpolation techniques. Curves generating from these techniques pass through the given points. Another alternative to create curves is to use approximation technique, which produces curves that do not pass through the given data points. These given points are used to control the shape of the resulting curves. Bézier and B-spline curves are examples based on approximation techniques.

Cubic Bézier curves are defined using four control points. Two of these are the end points of the curve, while the other two effectively define the gradient at the end points. These two points control the shape of the curve. The curve is actually a blend of the control points. This is a recurring theme of approximation curves; defining a curve as a blend of the values of several control points.

Bézier curves have many interesting properties: They (i) satisfy convex hull property and (ii) interpolate starting and ending control points.

In the early stages, Bézier curve representation is used to design the outer shape of some Renault cars. These days it is widely used in computer graphics and geometric modeling. Cubic Bézier curves are described by a parametric equation. Four control points are used to describe a specific instance of a curve. Bézier curves are used in a variety of different ways in computer graphics. Many scalable fonts such as "True Type" use Bézier curves to define the outline of each character. The page description languages

PostScript and PDF also have curve operators, which utilize Bézier curves. Bézier curves are so common for several reasons since implementation is relatively straightforward when compared to other forms of parametric curves. The use of control points to specify the shape of the curve is a major factor in this. Other properties make them easy for a user to work with and still providing good results.

The main objective of this study is the generation of Sinhala characters using cubic rational Bézier curves. When generating a character it is broken down to several segments depending on the curvature. For each of the segments, a Bézier curve was created.

Another goal of this study is to create a database, which consists of control points and their weights for each segment to which Sinhala characters were broken down.

The development of a computer program to read database and generate Sinhala characters is also an aim of this study. The computer program should display the character when the user requests it by selecting the character from a menu.

Chapter 2

PRELIMINARIES

Traditionally curves were created using either interpolation techniques or approximation techniques. In this study, We use approximation techniques, which produce curves that do not pass through the given data points. These given points are used to control the shape of the resulting curves. Bézier curves are examples based on approximation techniques.

This study is concerned with the generation Sinhala characters using cubic rational Bézier curves. In this section we discussed the assumptions, properties, constraints and theories used in the modeling.

When generating characters, they are broken down to sections depending on the curvature and for each section a rational Bézier curve was created.

2.1 Nonrational Bézier Curve

The form of the nonrational Bézier curve is as follows.

$$\underline{P}(u) = \sum_{i=0}^{n} \underline{P}_i B_{i,n}(u) \; ; \qquad 0 \leq u \leq 1$$

Here $\underline{P}(u)$ denotes the vector of points on Bézier curves. $\underline{P}_i$ depicts the control points and $B_{i,n}$ are the blending coefficients. Blending coefficients are calculated as follows.

$$B_{i,n}(u) = \binom{n}{i} u^{i}(1-u)^{n-i} \qquad \binom{n}{i} = \frac{n!}{i!(n-i)!}$$

Here u varies from 0 to 1 and different values of u gives different points on the Bézier curve. Shape of a nonrational Bézier curve depends on control points.

Since a nonrational Bézier curve cannot represent a conic section such as circle exactly, rational Bézier curves are more suitable to generate Sinhala characters. In rational Bézier curves, suitable weights are attached to control points which gives more flexibility to the curve designer to obtain the required shape.

2.2 Rational Bézier curve (n th order)

The general form of the n^{th} order rational Bézier curve is as follows. Here it uses $n+1$ control points.

$$\underline{P}(u) = \frac{\sum_{i=0}^{n} w_i \underline{P}_i B_{i,n}(u)}{\sum_{i=0}^{n} w_i B_{i,n}} \quad , \quad \text{where} \quad 0 \le u \le 1$$

Assign each control point a "weight, w_i ", then multiply the coordinates by w_i, and treat $w_i P_i$ as another coordinates. Here if we apply uniform weights, this is identical to nonrational Bézier curve. This is because $\sum B_{i,n}(u)=1$ for any u.

Form of the Cubic Rational Bézier curve

$$\underline{P}(u) = \frac{w_o(1-u)^3\underline{P}_0 + 3w_1u(1-u)^2\underline{P}_1 + 3w_2u^2(1-u)\underline{P}_2 + w_3u^3\underline{P}_3}{w_o(1-u)^3 + 3w_1u(1-u)^2 + 3w_2u^2(1-u) + w_3u^3}$$

where $0 \leq u \leq 1$

2.3 Properties of Cubic Rational Bézier curves

- Satisfy convex hull property, which is useful when displaying or clipping the curve.
- Interpolate starting and ending control points.
- Starting tangent $=3(\underline{P}_1-\underline{P}_0)$, (i.e. tangent to the Bézier curve at start is parallel to the line connecting first two control points).
- Ending tangent $=3(\underline{P}_3-\underline{P}_2)$, (where P_2 & P_3 are last two control points).
- The curve is continuous and infinitely differentiable.
- All the weights, w_i can not be simultaneously zero.
- The weights affect the shape in the interior.
- Bézier curve exhibits the variation diminishing.
- Bézier curve may have locally linear segments embedded in it, which is a useful design feature.
- The curve is symmetric with respect to u and (1-u). This means that the sequence of points defining the curve can be reversed without change of curve shape.
- A series of Bézier curves can be designed to satisfy zero order and first order continuity by making the control polygons to satisfy the same.
- Invariance under affine transformation.

Some of these properties can be illustrated as follows.

Invariance under affine parameter transformations

Mathematical expression: Consider Bézier curve $\underline{P}(u)$ defined over an interval [0,1]. The identical curve can be defined using a parameter t defined over the interval [*a*,*b*] by using the linear transformation $t=(1-u)a+ub$. Thus

$$\underline{P}(u) = \sum \underline{P}_i B_{i,n}(u) \quad , \qquad 0 \le u \le 1$$

$$= \sum \underline{P}_i B_{i,n}(\frac{t-a}{b-a}) \quad , \qquad a \le t \le b$$

where

$$B_{i,n}(u) = \frac{n!}{(n-i)!(i!)}(1-u)^{n-i}u^i$$

Geometric meaning: This means that if we transform the interval [0,1] to [*a*, *b*], the Bézier curve stays invariant. Since the transition from the interval [0,1] to [*a*, *b*] is an affine map, we say that Bézier curves are invariant under affine parameter transformations.

(Reference:www.math.hmc.edu/faculty/gu/math142/mellon/Application_to_CAGD/Bézier curves/ Properties.html)

Common usage: Because of this proposition, we can generate Bézier curves defined over the *interval* [*a*, *b*]. However, it is usually convenient and saves calculations if we use Bézier curves *P(u)* defined over [0,1], wherever possible.