

**UNCERTAINTY ANALYSIS IN MASS AND PRESSURE  
MEASUREMENTS**

by

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The work described in this thesis was carried out by me under the supervision of Mr.K.A.Gunasoma and Dr.Menaka Liyanage and a report on this has not been submitted in whole or in part to any university or any other institution for another Degree/Diploma

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## List of Abbreviations

NML	National Measurement Laboratory
CGPM	General Conference of Weights and Measures
SI	International System
VIM	International Vocabulary of basic and General Terms in Metrology
CIPM	Comite International des Poids et Mesures
BIPM	Bureau International des Poids et Mesures
IEC	The International Electrotechnical Commission
IFCC	International Federation of Clinical Chemistry
IUPAC	International Union of Pure and Applied Chemistry
IUPAP	International Union of Pure and Applied Physics
OIML	International Organization of Legal Metrology
ISO	International Organization for Standardization
GUM	Guide to the expression of Uncertainty in Measurement
JCGM	Joint Committee for Guides in Metrology
MCM	Monte Carlo Method
PDF	Probability Density Function
NPL	National Physical Laboratory
MSL	Measurement Standards Laboratory

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## ABSTRACT

A measurement result is complete only when accompanied by qualitative statement of its uncertainty. The measurement is required in order to decide if the result is adequate for its intended purpose and to ascertain if it is consistent with other similar results. Without such a measure it is also impossible to judge the fitness of the value as a basis for making decisions relating to health, safety, commerce or scientific excellence.

Thus aim of this project is to propose better uncertainty methods for Measurement Units, Standards and Services Department (MUSSD) by analyzing the uncertainty methods used in various National Measurement Laboratories (NMLs) in mass and pressure measurements. Two internationally accepted master documents for evaluating and expressing uncertainty in measurement namely “The Guide to the Expression of uncertainty in measurement” and its supplement are used as a guide.

Uncertainties in mass measurements are calculated for 1g -500g by the method described in GUM, by the method use in National Physical Laboratory (NPL) in India and also by the method of Monte Carlo (MCM) using MATLAB codes.

Uncertainties in pressure measurements are calculated by fitting a linear model using least square method and the method that used in Measurement Standards Laboratory (MSL), New Zealand. Both direct evaluation method and Monte Carlo Method (MCM) are used for the evaluation of uncertainty of linear model which gives almost similar results.

The uncertainty analysis shows that Monte Carlo Method (MCM) and the method used in MSL give better results for mass and pressure measurements respectively, which can be recommended for Measurement Units, Standards and Services Department.

## Chapter 1

### INTRODUCTION

#### 1.1 Importance of Measurements

Metrology is the name given to the science of measurements. Measurements have a great impact on our everyday lives, playing important roles in fields such as trade, production, agriculture, health sector, energy production and distribution, telecommunications, transport, construction, safety, industry and research and development. A credible measurement system is therefore vital for the overall well-being of society. In addition to its societal impact, science of measurements contributes significantly to the economic situation of a country, as witnessed when determining the global value of products and services whose value can only be assessed through measurements. Harmonization of the requirements governing this system not only provides an important basis for measurement credibility, but also serves to promote international trade through the elimination of technical barriers.

In order to investigate the results of measurements on a consistent basis, it is necessary to establish

- Standards of units of length, weight, time, temperature etc.
- Traceability or unbroken chain of comparisons.

Other than the above two properties, uncertainty of measurements plays a main role in metrology. An expression of a measurement is incomplete unless it includes a statement of the associated uncertainty. (ISO, 1995, p4) An estimate of uncertainty is required in order to decide if the result is adequate for its intended purpose and to ascertain if it is