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ADOPTATION OF SCHEFF'S PROCEDURE TO TEST OF EQUALITY BETWEEN SETS OF COEFFICIENTS IN TWO LINEAR REGRESSIONS WHEN DISTURBANCE VARIENCES ARE UNEQUAL

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Abstract: *Using Scheffe's test procedure a test is proposed to test the equality of coefficients of two linear regressions.*

1. Introduction

Scheff'e (1943, 1944) considered the problem of means of two samples when the variances of the samples are unequal and proposed a statistic for exact confidence intervals for the difference of the two means. To obtain the result he transformed the data by choosing transformation coefficient so as to minimise the expected length of the confidence intervals of the difference of two means. Transforming our data using a similar transformation and then applying usual test procedure we also can obtain test statistic to test the equality of coefficients of two regressions.

2. The F Test

The conventional F test is concerned with testing the validity of the exact linear restrictions $R\beta = r$ on the parameters of the model

$$Y = X\beta + e, \quad e \sim N(O, \sigma^2 I_T) \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where Y and e are Tx1 stochastic matries, β is a kxl matrix of parameters and X, R and r are respectively Txk, mxk and mxl matrices of fixed numbers.

If we assume that X has rank k and R has rank m the Ordinary Least Square Estimators of the parameters of the model (1) are given by

$$\hat{\beta} = (X^1X)^{-1} X^1Y \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Since $e \sim N(O, \sigma^2 I_T)$ it follows that

$$\hat{\beta} \sim N[\beta, \sigma^2 (X^1X)^{-1}]$$

and $R\hat{\beta} - r \sim N[R\beta - r, \sigma^2 R (X^1X)^{-1} R^1]$

Under the null hypothesis H_0 of $R\beta = r$

$$R\hat{\beta} - r \sim N[O, \sigma^2 R\ (X^1X)^{-1}R^1]$$

It follows that $(R\hat{\beta} - r)^1 [\sigma^2 R\ (X^1X)^{-1}R^1]^{-1} (R\hat{\beta} - r) \sim X^2_m$ ....(3)

Using the properties of the ordinary least square regression we have

$$\frac{\hat{e}^1 \hat{e}}{\sigma^2} \sim X^2_{T-k} \qquad \text{...............(4)}$$

where ê is TXI vector of ordinanry least squres residuals from the model (1). Since the statistics (3) and (4) are statistically independent we have

$$\frac{(R\hat{\beta} - r)^1 [R\ (X^1X)^{-1}\ R^1]\ (R\hat{\beta} - r) / m}{\hat{e}^1 \hat{e} / (T-k)} \sim F_{m,T-k} \qquad \text{..................(5)}$$

3. The Result

Consider the model

$$Y_1 = X_1\beta_1 + e_1\ ,\ e_1 \sim N(O,\ \sigma_1{}^2 I_{T1}) \text{ ....(6)}$$

$$Y_2 = X_2\beta_2 + e_2\ ,\ e_2 \sim N(O,\ \sigma_2^2\ I_{T2})\ ,\ E\ (e_1 e_2^1) = O,$$

Where Y_i and X_i are $T_i$xl and $T_i$xk observations matrices with X_i having full column rank, β_i is a kxl coefficient matrix and e_i is $T_i$xl disturbance matrix, for i=1, 2.

Let the model for the t th observation be

$$Y_{1,t} = X_{1,t}\beta_1 + e_{1,t}\ ,\ t = 1, 2, \text{............}, T_1 \qquad \text{..................(7)}$$

$$Y_{2,t} = X_{2,t}\beta_2 + e_{2,t}\ ,\ t = 1, 2, \text{............}, T_2 \qquad \text{..................(8)}$$

Without loss of generality we can assume the number observations in the first sample to be greater than the number of observations in the second sample- i.e. $T_2 < T_1$

Define $$V_t = Y_{2,t} - \left(\frac{T_2}{T_1}\right)^{\frac{1}{2}} Y_{1,t} + \frac{1}{(T_1T_2)^{\frac{1}{2}}} \sum_{i=1}^{T_2} Y_{1,i} - \bar{Y}_1 \qquad \text{...............(9)}$$

$$t, = 1, 2, \ldots, T_2$$

where $\bar{Y}_1$ is the mean value of the observations in Y_1

Substituting for $Y_{1,t}$ and $Y_{2,t}$ in (9) from (7) and (8) we have

$$V_t = X_{2,t}\beta_2 + e_{2,t} - \left(\frac{T_2}{T_1}\right)^{\frac{1}{2}}\left[X_{1,t}\beta_1 + e_{1,t}\right]$$

$$+ \frac{1}{(T_1T_2)}^{\frac{1}{2}}\sum_{i=1}^{T_2}\left[X_{1,i}\beta_1 + e_{1,i}\right] - \left[\bar{X}_1\beta_1 + \bar{e}_1\right] \text{.................}(10)$$

Where $\bar{X}_1$ is the row vector of columns means of X_1 matrix - Rearranging the terms in (10) we have.

$$V_t = P_t\beta_2 + Q_t\beta_1 + U_t \text{ ,..........................}(11)$$

Where $P_t = X_{2,t}$, $Q_t = -\left(\frac{T_2}{T_1}\right)^{\frac{1}{2}} X_{1,t} + \frac{1}{(T_1T_2)}^{\frac{1}{2}}\sum_{i=1}^{T_2} X_{1,i} - \bar{X}_1$

and $U_t = e_{2,t} - \left(\frac{T_2}{T_1}\right)^{\frac{1}{2}} e_{1,t} + \frac{1}{(T_1T_2)}^{\frac{1}{2}}\sum_{i=1}^{T_2} e_{1,i} - \bar{e}_1$, $t = 1, 2, \ldots\ldots, T_2$

For this transformed model (11) we have

$$E\,U_t = O \quad , \quad \text{for } t = 1, 2, \ldots\ldots\ldots\ldots\ldots, T_2$$

$$E\,U_t^2 = \sigma_2^2 + \left(\frac{T_1}{T_2}\right)\sigma_1^2, \quad \text{for } t = 1, 2\ldots\ldots\ldots\ldots\ldots, T_2$$

and $E\,U_t\,U_s = O$, for $t \neq s$; $t, s = 1, 2 \ldots\ldots\ldots\ldots\ldots\ldots T_2$

Therefore the transformed model (11) satisfies the conditions of the classical linear model.

Collecting all the observations we can write the model (11) as follows:

$$V = P\beta_2 + Q\beta_1 + U \qquad \text{.....................}(12)$$

Where V is the T_2 X I vector of observations on the transformed dependent variable, and P and Q are T_2Xk matrices of observations on the transformed independent variables, β_1 and β_2, are kX1 coefficient vectors and U is the T_2X1, vector of transformed disturbances.

Write $$V=ZB+U \qquad \text{..............(13)}$$

where $Z=[P\ Q]$ and $B=\begin{bmatrix}\beta_2\\ \hline \beta_1\end{bmatrix}$

To test the hypothesis $\beta_1=\beta_2$ we can use the result (5) in section (2) with $R=[I_K - I_K]$ and $r=O$.

Then it follows that under the mull hypothesis $\beta_1=\beta_2$ the statistics

$$\frac{(\hat{\beta}_1-\hat{\beta}_2)^1\ [A_{11}-A_{21}-A_{12}-A_{22}]^{-1}\ (\hat{\beta}_1-\hat{\beta}_2)\ /\ k}{\hat{U}^1\hat{U}\ /\ (T_2-2\ k)} \qquad \text{..............(14)}$$

is distributed as F distribution with k and T_2–2k degrees of freedom, where $\hat{\beta}_1$ and $\hat{\beta}_2$ are the OLS estimaors of β_1 and β_2, $\hat{U}$ is the OLS residuals from the model (12) and A' s are given by the following expression.

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} P^1P & P^1Q \\ Q^1P & Q^1Q \end{bmatrix}^{-1}$$

We may therefore use the statistic (14) as a small sample test of equality between sets of coefficients in two linear regressions.

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