MODELING OF GRAVITY ANOMALIES USING A FOURIER ITERATIVE TECHNIQUE

BY

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The work described in this thesis was carried out by me under the supervision of Professor D. A. Tantrigoda and a report on this has not been submitted in whole or part to any University or other institution for another degree.

....

D.S.Rodrigo

I certify that the above statement made by the candidate is true and that this thesis is suitable for submission to the University for the purpose of evaluation.

....

Professor D. A. Tantrigoda.

I certify that the candidate has incorporated all the corrections, amendments and additions recommended by the examiners.

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Professor D. A. Tantrigoda.

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ABSTRACT

A computationally efficient method of modeling gravity anomalies using the well known Parker series is presented. This is a modification of an already existing method that is applicable only for modeling of gravity anomalies caused by sedimentary basins. The modified version which is referred to as PITM method can be used to model gravity anomalies caused by sedimentary basins, igneous intrusions and geological faults. Both two and three dimensional modeling can be performed using this method. Most important feature of the PITM method is that it provides enhanced maneuverability to the person who carrying out the modeling. This feature helps to model bodies with geologically realistic shapes which have applications in oil and mineral exploration industry as opposed to unrealistic bodies which are mere mathematical artifacts. This method can also be used to model gravity anomalies in terms of bodies subject to constraints and also gravity gradient data after slight modifications.

The study commences with a comprehensive investigation of the Parker series which includes its detailed derivation following an outline given by the author of the original publication, studying the convergence of the series and estimating the relative magnitudes of its terms which facilitate the justification of truncation of the series appropriately.

CHAPTER 1

Introduction

1.1 Background

World of philosophers and mathematicians and the world of work created by the artisans existed as two different entities until the intervention of Galileo Galilee in 16^{th} century(Hawking,2002). Galileo who held the prestigious position of Professor Mathematics at the Padova university in Italy made an attempt fuse these two worlds which lead to the birth of modern science. Model formulation and testing the models became an important aspect of modern science and mathematics has played an immensely important role in this regard. In model formulation a set of assumed physical properties of a system that is under investigation are related to observations and the tool that is used to establish this relationship is mathematics. These relationships very often are non-linear and can be expressed as mathematical equations such as partial differential equations or as integral equations. Determination or estimation of the physical properties can be carried out by solving these equations which is again a formidable task in mathematics. In certain cases using simplifying assumptions, such relationships can be made linear and they can be expressed as a set of linear equations. Such equations can be solved using the linear inverse theory. Further, mathematics has played a crucial role in understanding the limitations of the solutions and also how to interpret the results in real world scenarios.

Geophysics has been one of the branches of science whose rapid development is very much indebted to mathematics. Seismology and potential fields play an important role in both pure and applied or exploration geophysics. In seismology propagation of elastic waves inside the Earth is used to understand the layered structure of the Earth and this is a formidable mathematical task. Study of potential fields initiated by Newton and Laplace in the past has today become an important and inexpensive exploration tool in geophysics. Potential field studies include analysis and modeling of gravity and magnetic anomalies.

It is possible to extract the part of the gravity field produced by Earth structures with densities different to those of surrounding rocks(Nettleton,1940,Dobrin,1976). These gravity effects can be considered as anomalies of the gravity field of the Earth assumed to be homogeneous. Mathematical methods are available to compute the gravity anomalies caused by complicated geological shapes. Estimation of the shape and size of a geological structure causing an anomaly or solving the inverse problem in gravity has become an important task in exploration geophysics, especially in view of its usefulness in search of hydrocarbons. One of the most computationally efficient methods of solving the forward problem in gravity has been presented by Parker (1972). Present study revolves around an investigation of the possibility of using this method to solve the inverse problem in gravity so that its computational efficiency can be exploited in the modeling process.

1.2 Aim and Objectives of the Study

The aim of the study is to investigate the possibility of using the Parker series to solve the inverse problem in gravity. Objectives of the studies are to

1. perform a detailed derivation of Parker series starting from the outline provided by Parker (1972).
2. investigate the convergence of the Parker series,
3. study the relative magnitudes of terms in the Parker series,
4. critically review the existing method which uses the Parker series to solve the inverse problem in gravity,
5. modify and improve the existing method so that it could be applied to model gravity anomalies caused common geological structures,
6. investigate the success of the modified method to model simulated gravity anomalies due to known structures,
7. investigate the possibility of using the modified method to model observed gravity anomalies due to real geological structures .
8. investigate the possibility of using the modified method to model gravity gradiometry data (Today there is a trend of measuring gravity gradients instead gravity in air-borne surveys).

1.3 Overview of Chapters

This thesis comprises six chapters and the first chapter has been devoted to provide the basic objectives of the study together with a glimpse its background.

The basic mathematical tool that is used in this study is the Fourier transforms together with discrete Fourier transforms. In the Parker series gravity anomaly of a body of varying thickness is expressed as a summation of Fourier series of various powers of the thickness of the body measured at nodes of a regular rectangular grid. In this case it is necessary to find the Fourier transform of a set of discrete data and therefore discrete Fourier transforms have to be used. There are several important aspects of discrete Fourier transforms that one has to be careful when using them and the chapter 2 provides a comprehensive discussion of these aspects which includes aliasing, Nyquist frequency, truncation error, Gibb's phenomena, even and odd symmetries of real and imaginary parts of a Fourier transform of a real function and frequency domain filtering.

Chapter 3 gives a review of techniques available for gravity modeling. It starts with an overview of the gravity field of the Earth and the definition of gravity anomaly. Then the brief discussion on the two most important and widely used modeling techniques namely non-linear-optimization and linear inverse theory have been briefly discussed.

Chapter 4 is devoted for a comprehensive discussion of the Parker series. Based on an outline of a derivation given by Parker (1972) and also using some information obtained through personal communication with the author, Prof. R.L. Parker, a detailed derivation of the series has been carried out. To use this series to solve the inverse problem in gravity it is very important to understand how rapidly the series converges and relative magnitudes of the terms as we truncate the series after the consideration of a certain number of terms. A comprehensive discussion of these aspects has been provided in this chapter.

Chapter 5 is mainly on the investigation of the possibility of using the Parker series to model gravity anomalies. First a critical evaluation of the existing method of solving the inverse problem using the Parker series is given. Then a modification of this method removing some of its shortcomings and enhancing its applicability to model gravity anomalies produced more geological features is presented.

The modified method was subjected to a series of numerical experiments to examine its capability to model gravity anomalies due to sedimentary basins, igneous intrusions, geological faults. The investigation involved a computation of the gravity anomaly due to a hypothetical geological structure of known dimensions and then use the method formulated in the study to model this gravity anomaly. Then the dimensions of the structure that is obtained from modeling is compared with known dimensions of the hypothetical structure used for the forward calculation to ascertain how successful the modeling process is. Then