

# A self-tuning Firefly algorithm to tune the parameters of Ant Colony System (ACSFA)

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## Abstract

Ant colony system (ACS) is a promising approach which has been widely used in problems such as Travelling Salesman Problems (TSP), Job shop scheduling problems (JSP) and Quadratic Assignment problems (QAP). In its original implementation, parameters of the algorithm were selected by trial and error approach. Over the last few years, novel approaches have been proposed on adapting the parameters of ACS in improving its performance. The aim of this paper is to use a framework introduced for self-tuning optimization algorithms combined with the firefly algorithm (FA) to tune the parameters of the ACS solving symmetric TSP problems. The FA optimizes the problem specific parameters of ACS while the parameters of the FA are tuned by the selected framework itself. With this approach, the user neither has to work with the parameters of ACS nor the parameters of FA. Using common symmetric TSP problems we demonstrate that the framework fits well for the ACS. A detailed statistical analysis further verifies the goodness of the new ACS over the existing ACS and also of the other techniques used to tune the parameters of ACS.

## 1 Introduction

The collective behavior of natural insects - ants, bees, fireflies and termites mimic the problem solving capabilities of the swarms [10, 13, 4]. These capabilities were adopted in various heuristics and meta-heuristics to solve difficult optimization problems. Each meta-heuristic has a real world inspiration of optimization. As such, the main inspiration for the ant colony system algorithm is the natural food finding strategy of ants. Ants are capable of finding the shortest path from the food source to their nests. A chemical inside them called pheromone is the reason for this optimized behavior. Following this real world strategy, the ant colony system algorithm was developed by Marco Dorigo et al.[6] to suit for path optimization problems. Initially the algorithm was developed to solve the TSP. This original implementation supports the hypothesis that the ant colony algorithm is successful in finding the shortest path for the TSP.

Similar to any meta-heuristic, the ant colony system also has algorithm specific parameters. The initial implementation used the trial and error method to find the best collection of parameters. The values of the parameters depend on the problem at hand and hence at each instance, the most suitable parameter set for the problem should be evaluated. The

task of parameter tuning again is an optimization where the optimal parameter set will give the optimum performance.

Though the initial study relied on trial and error, Dorigo had stated some important features of parameters such as pheromone behavior, the number of ants and how they affect the performance of the algorithm [6]. The original paper discusses the ranges for different parameters so that an idea about the distribution of each parameter is given. The results were in an encouraging position ascertaining that the algorithm is successful in solving the TSP. Instances from TSPLIB and randomly generated TSPs' were used to evaluate the algorithm.

Since the original ACS works well with pre-tested parameter values, finding approaches to set better parameters without trial and error can improve the performance of the algorithm. The best way is to consider setting of parameters as another optimization problem. The same matter of tuning parameters of an ACS is considered by many other researchers. Yet none was successful in maintaining fully parameterless environment. An Adaptive Parameter Control Strategy for Ant Colony Systems by Zhi-Feng Hao et al. is a study carried out to enhance the performance of the ant system solving the TSP [8]. Although it has been mentioned as the ant colony system, they have used the ant system for the study. The particle swarm optimization (PSO) has been used to optimize the parameters of the ant system (PSOACS). The parameters of the ant system were considered as a whole where one particle represents approximation for the set of parameters. The number of particles is the same as the number of ants, and once ants complete a tour the PSO will update the parameters. The conclusions are based on the performance of the new algorithm and the existing ACS. The results support the conclusions, but the following matters were not addressed. Although the study focused on a parameter tuning technique, the effects of the method towards the parameters were not mentioned. Basically the study was focused on improving the TSP results of the existing approach. For both PSOACS and original ACS, the results obtained conflict with the optimal results given in the TSPLIB. The reason may be the differences in the implementations. Also in PSOACS, the equations used are not clear enough to get an idea about the new algorithm.

In Evolving Ant Colony Optimization, another research by Hozefa M. Botee and Eric Bonabeau, they have made the use of genetic algorithm (GA) to evolve the best set of parameters [3]. Here also, one parameter set represents an individual of the genetic algorithm. However the implementation was tested only over two TSP instances. The results were encouraging, but the parameters of the ACS depend on the selection of the parameters of GA such as crossover and mutation probability.

Apart from meta-heuristics, machine learning techniques have also been used to tune parameters of ACS. For example Ayse Hande Erol et al. in their research, have used artificial neural networks (ANN) to find the best parameter set for ACS solving a given TSP [7]. The research focuses on only two parameters,  $\alpha$ : pheromone decay parameter and  $\beta$ : the relative importance of the pheromone vs. distance. Initially the ACS-ANN hybrid algorithm runs with different  $\alpha$  and  $\beta$  values for 50 times. These parameters work as the inputs to the ANN. Then ANN predicts the best parameter values for a given TSP instance. The hybrid algorithm was tested with several TSP instances from the TSPLIB [21]. The results support the hypothesis that the hybrid algorithm performs well in finding better parameter values. However the study focused on tuning only two parameters. The information about the ANN

such as the training methods, weights and their effects are not mentioned in the research.

As such there are other researches which have been conducted to find better parameter sets for the ACS in solving TSP [9, 20]. But as a whole, all these methods rely on another algorithm or technique, where it again contains its own parameters. Therefore the performance of the ACS again depends on the parameters of the used algorithm. To overcome this issue we have designed an algorithm with the help of the firefly algorithm and the self-tuning framework for optimization algorithms proposed by Xin-She Yang, to optimize the parameters of the ACS algorithm solving symmetric TSP instances. Xin-She Yang et al., in the implementation of the self-tuning framework have used the firefly algorithm to apply the framework to tune FA's parameters [19]. In their framework, the problem solved by the optimization algorithm and the parameter set of the algorithm both were considered as a single problem. The framework was initially tested for the firefly algorithm and proved its capability of tuning parameters of itself (FA). In a research done by M.K.A. Ariyaratne et al., use this self tuning framework combined with the firefly algorithm to solve nonlinear equations [2]. They have solved univariate nonlinear equations having complex roots with the help of a modified firefly algorithm. To find the best parameters values, the self tuning framework was implemented on the firefly algorithm. In their research, a firefly carries an approximation for a root as well as approximations to the parameter values. Finally as the output, they receive best approximations for the roots in a given range as well as best approximations for the parameter values. The research again confirms the powerfulness of the self tuning framework in optimizing parameters.

The significance of this research lies in the potential of the developed ant colony optimization algorithm for the TSP with the self tuning framework to optimize the parameters. Despite the recent advancements in the field of route optimization and parameter optimization, the following issues have not been addressed by other researchers (See Table ?? for details of previously applied approaches) where our research has accomplished.

- None of the existing systems are capable of providing virtually parameter-free environments for the ant colony systems to solve TSPs.
- In most of the researches, parameter optimization is done using another meta-heuristic algorithm whose parameters should be manually selected.
- None of the approaches have used a designed framework for parameter optimization.

Author	Year	Parameter Optimization method	# of TSP problems tested	Advantages	Limitations
Hozefa M. Botee and Eric Bonabeauy	1998	GA	2	GA optimizes the parameters of ACS	Only some parameters of ACS were optimized. The parameters of GA should be manually updated.
Raed Abu Zaitar and Hussein Hiyassat	2005	GA	8	GA optimizes the parameters of ACS	Only two parameters of ACS ( $\alpha$ and $\beta$ ) were updated in the first phase.
D. GÃşmez-Cabrero and D. N. Ranasinghe	2005	PSO	24	PSO optimizes the parameters of ACS	The parameters of PSO should be manually updated. High computational overhead.
Zhi-Feng Hao et al.	2006	PSO	10	PSO optimizes the parameters of ACS	The parameters of PSO should be manually updated.
Ayse Hande Erol et al.	2012	Artificial Neural Networks (ANN)	16	ANN optimizes the parameters of ACO	Only some parameters of ACO were optimized.

Table 1: Previous work on parameter optimization of Ant Systems

GA-Genetic Algorithms, PSO-Particle Swarm Optimization, ACS-Ant Colony Systems, ACO-Ant Colony Optimization

In this paper, Yang’s self-tuning framework is studied, and a parameter selection strategy based on the firefly algorithm is developed. To deliver a better idea about the present work, the remainder of this paper is structured as follows. Section 2 provides some preliminaries related to the Ant colony system and the firefly algorithm. In section 3, we briefly describe the self-tuning framework for the firefly algorithm and how we adopt it to tune the parameters of the ACS when solving symmetric TSPs. There, we also present the hybrid ACS-FA. Section 4 points out the TSP examples, the parameters used and the results obtained from the new approach. To emphasize the goodness of the new algorithm, we compare the results with the original ACS and with some other parameter tuning approaches. Finally, we draw conclusions briefly in Section 5.

## 2 Preliminaries

### 2.1 Traveling Salesman Problem (TSP)

The Traveling Salesman Problem is one of the most intensively studied problems in computational mathematics which is simple to state but very difficult to solve [1]. The problem is NP hard, making it not computable in polynomial time [15]. The problem is about finding the shortest possible tour through a set of  $n$  cities/nodes so that each city/node is visited exactly once. A weighted graph  $G(N, E)$  can represent a TSP, where  $N$  represents the cities and  $E$  represents the set of edges connecting cities. There is a specific distance  $d$  for each  $(i, j) \in E$ . If  $d(i, j) = d(j, i)$ , it is known as a symmetric TSP where in an asymmetric TSP,  $d(i, j) \neq d(j, i)$  can occur. In our study we consider only the symmetric situation.

### 2.2 Ant Colony Systems (ACS)

Ants, the popular social insects normally live in colonies represent a highly structured distributed system. There are many ant species, some of which are blind. All ant species are known to be deaf [11]. Despite of these incapacibilities, ants are inbred with a strong indirect communication using a chemical produced within them. While foraging, ants lay the chemical; pheromone on the ground and follow the pheromone placed by other ants. The pheromones tend to decay over time and hence the ants in the colony will choose the path with high pheromone density at the moment. This pheromone communication allows the ants to find the shortest path from the food source to their nest. The optimized behaviors of real ants are based on implementing artificial ant colonies.

The ant colony systems is an example of an ant colony optimization method from the field of swarm intelligence, meta-heuristics and computational intelligence. Around 1990's Marko Dorigo introduced the idea of the ant system and later Dorigo and Gambardella introduce the ant colony system. In the original implementation, ACS was applied to solve the TSP [6]. Initially, ants in the artificial colony are positioned on random nodes/ cities. They travel from one node to another keeping the travel history in a data structure. The likelihood of selecting a node is based on the pheromone density of the cities laid by other ants, which in the algorithm known as the state transition rule. Once visiting a city, an ant lays an amount of pheromone using the local pheromone updating. Upon completing the tours by all ants, the cities belong to the globally best path again get updated with pheromones using global pheromone updating rule.

#### 2.2.1 State Transition Rule, Local and Global updating

State transition rule is responsible for an ant to find its next visiting city. Assume the ant is in the node  $r$ . It's next city  $s$  is determined by the equation 1.

$$s = \begin{cases} \arg \max_{u \in J_k(r)} \{[\tau(r, u)]^\theta \cdot [\eta(r, u)]^\beta\} & q \leq q_0 \\ S & \text{Otherwise} \end{cases} \quad (1)$$

where  $\tau(r, u)$  is the pheromone density of an edge  $(r, u)$ ,  $\eta(r, u)$  is  $[1/\text{distance}(r, u)]$  for TSP.  $J_k(r)$  is the set of cities that remain to be visited by ant  $k$  positioned on city  $r$ . The relative

importance of the pheromone trail and the heuristic information are represented by the parameters  $\theta$  and  $\beta$  ( $\theta, \beta \geq 0$ ).  $q$  is a random number uniformly distributed in  $[0, 1]$ ,  $q_0$  is a parameter ( $0 \leq q_0 \leq 1$ ), and  $S$  is a random variable from the probability distribution given by the equation (2).

$$P_k(r, s) = \begin{cases} \frac{[\tau(r, u)]^\theta \cdot [\eta(r, u)]^\beta}{\sum_{u \in J_k(r)} [\tau(r, u)]^\theta \cdot [\eta(r, u)]^\beta} & \text{if } s \in J_k(r) \\ 0 & \text{Otherwise} \end{cases} \quad (2)$$

ACS local and global updating happens according to the equation (3) and equation (4) respectively.

$$\tau(r, s) \leftarrow (1 - \rho) \cdot \tau(r, s) + \rho \cdot \Delta\tau(r, s) \quad (3)$$

where  $0 < \rho < 1$  is a parameter.

$$\tau(r, s) \leftarrow (1 - \alpha) \cdot \tau(r, s) + \alpha \cdot \Delta\tau(r, s) \quad (4)$$

where

$$\Delta\tau(r, s) = \begin{cases} (L_{gb})^{-1} & \text{if } (r, s) \in \text{global best tour} \\ 0 & \text{Otherwise} \end{cases} \quad (5)$$

$0 < \alpha < 1$  is the pheromone decay parameter and  $L_{gb}$  is the length of the globally best tour. In the original implementation, Dorigo et al. have given the set of parameter values obtained from the trial and error approach to suit the selected TSP instances.

The ACS grasped the attention of the world of optimization and hence many researches have been carried out to improve the algorithm as well as to check its ability over solving other optimization problems.

## 2.3 Firefly Algorithm (FA)

Firefly is a winged beetle commonly known as the lightning bug due to the charming light it emits. The light is used to attract mates or preys. Biological studies reveal many factors about fireflies' life style that are interesting [14]. Focusing on their flashing behavior, the firefly algorithm was developed by Xin-She-Yang in 2009 [18]. The algorithm basically assumes the following.

- Fireflies' attraction to each other is gender independent.
- Attractiveness is proportional to the brightness of the fireflies, for any two fireflies, the less brighter one is attracted by (and thus moves toward) the brighter one; however, the brightness can decrease as the distance increases; If there is no brighter one than a particular firefly, it moves randomly.
- The brightness of a firefly is determined by the value of the problem specific objective function.

As many meta-heuristics, the initial population for the particular problem is generated randomly. In FA also, the parameter set should be specified properly. After these initial steps, the fireflies in the population start moving towards brighter fireflies according to the following equation.

$$x_i = x_i + \beta(x_j - x_i) + \alpha(rand - 0.5) \quad (6)$$

where

$$\beta = \beta_0 \cdot e^{-\gamma r^2} \quad (7)$$

$\beta_0$  is the attraction at  $r = 0$ . The three terms in equation (6) represent the contribution from the current firefly, attraction between two fireflies and a randomization term respectively. The equation supports both exploitation and exploration.  $\alpha$  plays an important role in the randomization process, which is from Uniform or Gaussian distribution. To control the randomness, Yang has used  $\delta$ , the randomness reduction factor which reduces  $\alpha$  according to the equation (8).

$$\alpha = \alpha_0 \cdot \delta \quad \text{where } \delta \in [0, 1] \quad (8)$$

FA, as a newcomer in the world of meta-heuristics marked its remarkable capability of handling optimization problems. Yang et al. in 2013 introduced a framework for self-tuning algorithms and it was implemented with the firefly algorithm successfully [19]. The framework allows a meta-heuristic algorithm to solve a problem while optimizing its own algorithm specific parameters.

### 3 Self-tuning firefly algorithm optimizing Ant colony system's parameters (ACSFA)

In this research, the main aim is to tune the parameters of the ant colony system algorithm solving symmetric TSP problems while obtaining the shortest path for the selected TSP. FA as an outstanding performer, has used here in tuning the parameters of the ACS. Another reason is that the self-tuning framework can be easily implemented with firefly algorithm rather than directly on ACS, since the ACS solves a discrete problem (TSP) and the parameters are continuous in nature.

Regarding the ACS,  $\beta, \theta, \rho$  and  $q_0$  parameters have to be tuned. The FA also has several parameters such as  $\alpha, \gamma, \beta$  and the randomness reduction factor  $\delta$ . The main aim of the self-tuning concept is to find the best parameter settings that minimize the computational cost. When applying the self-tuning framework to the firefly algorithm solving a given optimization problem, both the problem domain and the parameter domain are considered as a single domain in solving the problem. The objective could be the objective of the problem.

In our case the FA is used to tune the parameters of ACS and the ACS solves a given TSP. The objective of the algorithms is to find the optimal solution of a given TSP instance. The problem of this study contains both parameters of ACS and FA. The pseudo code of the proposed ACSFA algorithm is presented in algorithm 1.

After initializing parameters and parameter ranges, inverse of a tour distance is set as the objective of the TSP. The problem is set as a minimization problem. The algorithm will build tours until a predetermined end condition is completed. As in the original ACS, ants are positioned on random starting nodes where each ant completes a tour using the state transition rule stated in [equation 1](#). While completing the tour ants will lay pheromones on the visited cities according to the [equation 3](#). Upon completing tours by all ants, a globally best tour will be identified and the cities belong to the globally best tour will be awarded with extra pheromone values according to the [equation 4](#). Apart from building tours, each ant also carries approximations of the parameters of both ACS and FA.

After completing a tour, all ants will work as fireflies. They now represent approximations of the parameters of FA and ACS. The parameters of ACS and FA will get updated using the self tuning firefly algorithm. The fireflies will move in the direction of better parameters/fireflies according to the [equation 6](#). At the end of each tour, the best parameter set will be detected.

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**Algorithm 1** : Pseudo code of the ACSFA

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1: Begin;
2: Initialize parameter values and ranges of parameter values to be tuned
3: Assign approximations of the parameters to be tuned to each ant (Firefly).
4: Define the objective function (I – Inverse of the distance)
5: while End condition do
6:   Begin Tour
7:     Position ants on starting nodes
8:     Build the tour while local pheromone update
9:   End Tour
10:  Do global pheromone update
11:  for  $i = 1 : n$  (all  $n$  ants) do
12:    for  $j = 2 : n$  ( $n$  ants) do
13:      if  $I_j < I_i$  then
14:        Move firefly  $i$  towards firefly  $j$  by using equation (6);
15:      end if
16:      Attractiveness varies with distance  $r$  via  $e^{-\gamma r^2}$  using equation (7);
17:      Evaluate new solutions and update parameter values;
18:    end for
19:  end for
20:  Rank the fireflies and find the current best (best parameter values);
21: end while
22: Post process results and visualization;
23: End

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## 4 Experimentation

The goal of this experiment is to analyze the performance of the ACSFA algorithm solving symmetric TSPs' while selecting the most suitable parameter values for ACS and FA. We aim to formulate the new algorithm with minimum number of user input parameters where all other parameters of the two algorithms are to be tuned while optimizing the TSP instances.

### 4.1 Parameter values and problem instances

The new algorithm is implemented to solve symmetric Traveling Salesman problems. 12 symmetric TSPs are selected for testing [21]. The parameters of ACS include:  $\beta, \alpha, \rho, q_0, m, \tau_0$  and  $\tau$ .  $\tau_0$  is based on the nearest neighbor heuristic. Pheromone density  $\tau$  depends on  $\alpha$  and  $\rho$  parameters. Since  $\alpha$  works only with the globally best ant, we initialize  $\alpha$  to be 0.1. Here, we consider  $\beta, \rho$  and  $q_0$  to be tuned by the self-tuning firefly algorithm. The range for these parameters are  $\beta \in [0 \ 8]$ ,  $\rho \in [0.5 \ 1]$  and  $q_0 \in [0.5 \ 1]$  which are large enough to select the best parameter values for many symmetric TSP instances. Apart from that, we tune the parameters of the FA. The parameters of firefly algorithm include  $\alpha, \beta, \gamma, \delta$  and number of fireflies. In equation 7, the brightness  $\beta$  depends on three main factors;  $\beta_0$  which we initiate to be 1,  $\gamma$ ; the light absorption coefficient and  $r$ ; the Cartesian distance between two fireflies which should be calculated during iterations. In equation 8, the value of  $\alpha$  get reduced with  $\alpha_0$  and  $\delta$ .  $\alpha_0$  is initialized to be 2.3. Therefore the value of  $\alpha$  depends on  $\delta$ . Hence the only factors to be tuned in FA are  $\gamma$  and  $\delta$ . For convenience, we will use  $FF\_alpha_0$ ,  $FF\_beta_0$ ,  $FF\_gamma$  and  $FF\_delta$  to indicate the parameters of FA. These parameters varies in the ranges  $FF\_gamma \in [0 \ 10]$  and  $FF\_delta \in [0.8 \ 1]$ . For further clarification a detailed list of the parameters used for the algorithms are presented in Table (2).

ACSFA		ACS		PSOACS	
Parameter	Value/ Range	Parameter	Value/ Range	Parameter	Value/ Range
$\alpha$	0.1	$\alpha$	0.1	$\beta$	[0 8]
$\beta$	[0 8]	$\beta$	2	$\rho$	[0.5 1]
$\rho$	[0.5 1]	$\rho$	0.1	$q_0$	[0.5 1]
$q_0$	[0.5 1]			$PSO\_Q_1$	2
$FF\_alpha_0$	2.3			$PSO\_Q_2$	2
$FF\_beta_0$	1				
$FF\_gamma$	[0 10]				
$FF\_delta$	[0.8 1]				

Table 2: Parameter values and ranges used for ACSFA, ACS and PSOACS

The new algorithm is implemented using MATLAB [12] and the experiment is conducted on a laptop with an Intel (R) Core (TM) i5-5200U CPU @ 2.20 GHz processor and 8GB memory. Since the speed relies on the programming language, structure and the type of the machine, the comparing algorithms were also implemented using the same environment. The

new algorithm is compared with the original ant colony algorithm (ACS) [6] and an adaptive parameter control strategy for ACO implemented using the PSO algorithm (PSOACS) [8]. The TSP instances and their best known solutions are indicated in Table (3).

<b>TSP Instance</b>	<b>Optimal Tour Length</b>
ulysses16	6859
bays29	2020
Oliver30	420
eil51	426
pr76	108159
kroA100	21282
lin105	14379
tsp225	3916
gil262	2378
lin318	42029
rat575	6773
rat783	8806

Table 3: Problem instances and the Optimal tour lengths found so far

## 5 Results

To accomplish the experimental comparison, we considered randomly selected 12 TSP instances from the TSPLIB, that have been presented in the Table (3) [21]. Table (4) presents the results. Each algorithm executed 10 times with each TSP instance to get the results. Results are formatted as the best TSP distance, the average, the worst and the average time taken by each algorithm. The obtained results illustrate the strength of the ACSFA over other two algorithms. The interesting factor is that, in ACSFA, results are better and most of the parameters are handled by the self tuning firefly algorithm.

TSP Instance	Algorithm	Best	Average	Worst	$t_{avg}$ (s)
ulysses16	ACSFA	6859	6891.2	6909	11.02
	PSOACS	6909	6909	6909	11.47
	ACS	6875	6891.2	6909	11.02
bays29	ACSFA	2026	2065	2085	36.58
	PSOACS	2028	2033.6	2036	36.87
	ACS	2038	2041.25	2042	33.30
Oliver30	ACSFA	421	425	426	22.27
	PSOACS	425	425.5	426	23.85
	ACS	426	428.83	434	19.77
eil51	ACSFA	428	432.6	438	67.30
	PSOACS	429	429.8	431	70.40
	ACS	430	434	439	60.16
pr76	ACSFA	108358	108474	108644	116.80
	PSOACS	108358	103755.8	110488	157.92
	ACS	110281	111342.6	112714	91.16
kroA100	ACSFA	21396	21390.71	21537	163.67
	PSOACS	21835	21874	21914	180.48
	ACS	22011	22703.16	23385	131.25
lin105	ACSFA	14412	14706.25	14860	211.09
	PSOACS	14492	14503.33	14571	194.43
	ACS	14844	15051	15353	153.10
TSP225	ACSFA	3978	4107.8	4283	519.57
	PSOACS	4009	4039.25	4059	611.89
	ACS	4077	4220.2	4308	395.20
gil262	ACSFA	2435	2448.16	2471	686.22
	PSOACS	2442	2472	2488	788.59
	ACS	2722	2872	2859	489.55
lin318	ACSFA	43061	43718.6	44192	1031.35
	PSOACS	43191	43737.6	44211	1156.92
	ACS	47960	48445	49427	595.66
rat575	ACSFA	7097	7420.33	7674	2952.31
	PSOACS	7189	7242.33	7346	3373.18
	ACS	7819	7877.5	7965	1609.39
rat783	ACSFA	10067	10226.2	10382	5998.23
	PSOACS	10540	10540	10540	6216.225
	ACS	10165	10491.25	10642	5143.26

Table 4: The best, average , worst performance and the average time of the algorithms

However, to prove the results in a convenient way, a statistical analysis is also conducted over the obtained results.

## 5.1 Statistical Analysis

A convenient statistical analysis was conducted to prove the validity of the results. The guidelines provided by Derrac et al. [5] were followed to perform the statistical analysis. However, here we have used parametric methods to conduct the statistical comparison. To test the hypothesis' related to the study, we have used ANOVA (Analysis of variance) technique [17]. ANOVA is useful when we need to do an experiment to conduct a comparison over more than 2 samples. Therefore, to compare the 3 Algorithms in terms of **error**, ANOVA with RCBD (Randomized Complete Block Designs) has been applied. The hypothesis tested here is:

$H_0$ : There is no any significant difference between 3 algorithms

$H_1$ : At least one algorithm is different from others

The results of the performed statistical test is as follows:

### Analysis for error

Method

Factor coding (-1, 0, +1)

Factor Information

Factor	Type	Levels	Values
Algorithm	Fixed	3	ACS, ACSFA, PSOACS
TSP	Fixed	12	Bays29, eil51, gil262, kroA100, lin105, Analysis lin318, Oliver30, pr76, rat575, rat783, TSP225, ulysses16

of Variance Source	DF	Adj SS	Adj MS	F - Value	P - Value
Algorithm	2	4043366	2021683	3.00	0.070
TSP	11	21614129	1964921	2.92	0.016
Error	22	14808697	673123		
Total	35	40466192			

S	R-sq	R-sq(adj)	R-sq(pred)
820.440	63.40%	41.78%	2.01%

Model Summary

Table 5: Results of ANOVA for the 'error'

P value of the algorithm field (0.07) is less than 0.1. Based on that, we reject  $H_0$  and the conclusion can be made as that there exist at least one algorithm which is significantly different from others at a 0.1 significance level. To find which algorithms are different from which, we have conducted a pairwise comparison over the algorithms. To conduct a pairwise comparison over the error, Tukey's method was applied [16]. The results were as follows.

## Tukey Pairwise Comparisons: Response = error, Term = Algorithm

Grouping Information Using the Tukey Method and 90% Confidence

Algorithm	N	Mean	Grouping
ACS	12	1016.75	A
PSOACS	12	366.67	A B
ACSFA	12	257.58	B

Means that do not share a letter are significantly different.

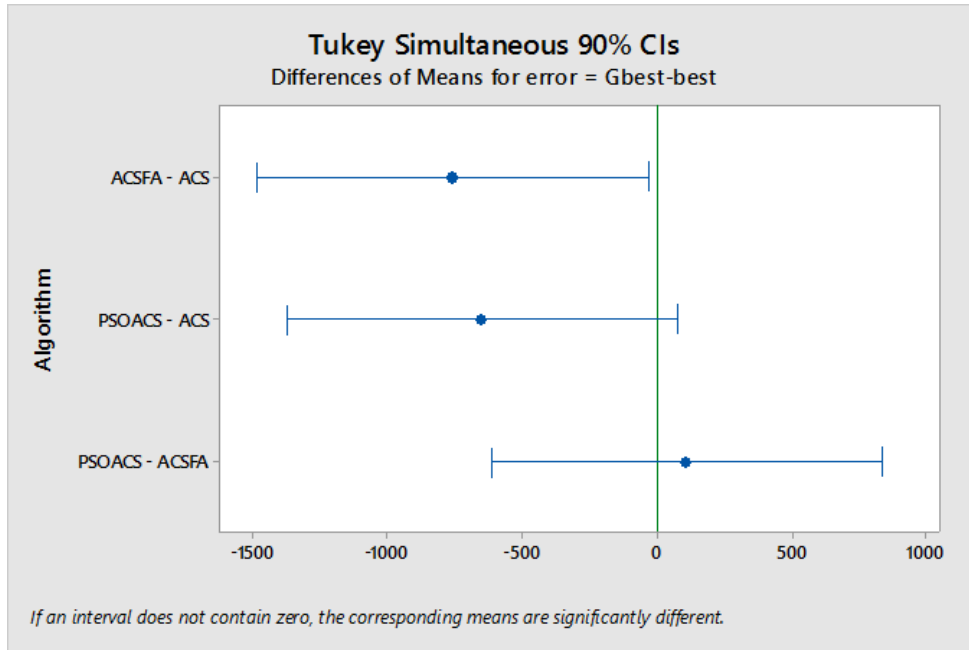


Figure 1: Tukey Pairwise Comparisons for ‘error’

The hypothesis used this time is as follows.

$H_0$ : There is no any difference between 2 algorithms

$H_1$ : There is a difference between 2 algorithms

The pairwise comparison on the error concluded that there is no any difference between ACSFA and PSOACS at 90% confidence level and there is a difference between ACSFA and ACS at 90% confidence level.

The same procedure was conducted considering the **average** and the **best** results of the algorithms for 12 TSP instances. Having P- values as 0.072 and 0.070, Analysis of variance in both cases supported to reject  $H_0$  concluding that at least one algorithm is significantly different from others at a 0.1 significance level. Pairwise comparisons were also conducted in both cases. The results are as follows.

### Tukey Pairwise Comparisons: Response = Average, Term = Algorithm

Grouping Information Using the Tukey Method and 90% Confidence

Algorithm	N	Mean	Grouping
ACS	12	19399.8	A
PSOACS	12	18525.5	A B
ACSFA	12	18163.5	B

Means that do not share a letter are significantly different.

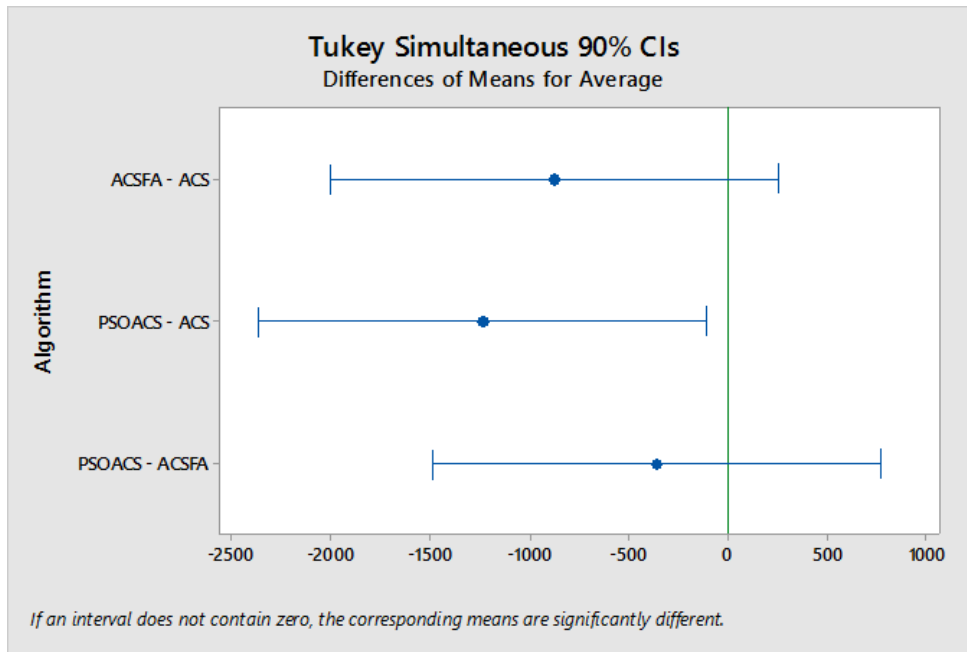


Figure 2: Tukey Pairwise Comparisons for 'average'

### Tukey Pairwise Comparisons: Response = Best, Term = Algorithm

Grouping Information Using the Tukey Method and 90% Confidence

Algorithm	N	Mean	Grouping
ACS	12	19137.3	A
PSOACS	12	18487.2	A B
ACSFA	12	18378.2	B

Means that do not share a letter are significantly different.

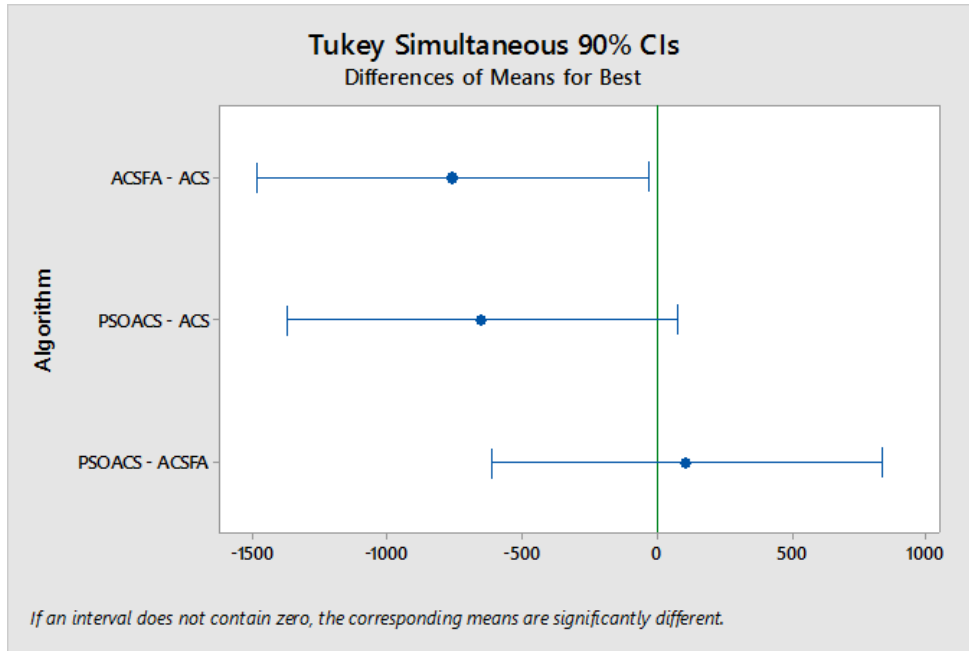


Figure 3: Tukey Pairwise Comparisons for ‘best results’

Both states support the conclusion that there is no any difference between ACSFA and PSOACS at 90% confidence level. Considering the best case, ACSFA and ACS appeared to be different at 90% confidence level.

Finally the analysis support the facts that the performance of ACSFA and PSOACS is equally strong where the performance of ACS is not as strong as ACSFA and PSOACS. But the results considering the best case, demonstrate that ACSFA outperforms other two algorithms. However although there is no significant difference between ACSFA and PSOACS from the statistical viewpoint, there exists a strong advantage of ACSFA over PSOACS: the ability of performing well without considering the selection of suitable parameter values for both ACS and FA.

## 5.2 Parameter Optimization

The statistical study emphasizes that there is no significant difference between ACSFA and PSOACS. But still ACSFA is better since it does provide a parameter-free environment to the user. The firefly algorithm with the self-tuning framework tunes the necessary parameters of ACS as well as FA. The figure 4 represents the evolution of parameter values of ACS over the iterations for the eil51 TSP instance. Here the mean value of each parameter for each iteration is calculated.

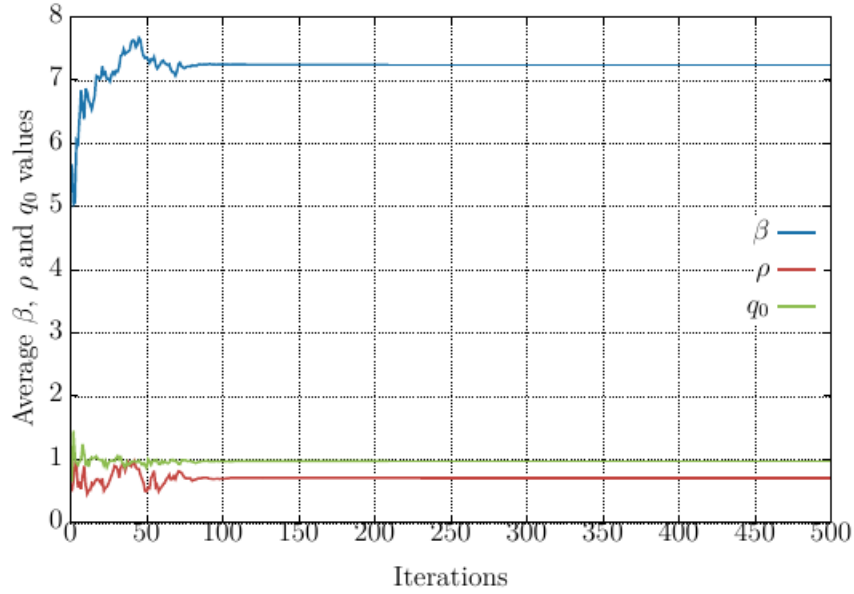


Figure 4: Variation of the average  $\beta, \rho$  and  $q_0$  values over iterations for eil51 TSP instance

It is necessary to see the behavior of the parameters of the FA as well. Figure 5 shows the evolution of the parameters of FA during iterations for the eil51 TSP instance. Here also, the mean value of each parameter for each iteration is calculated.

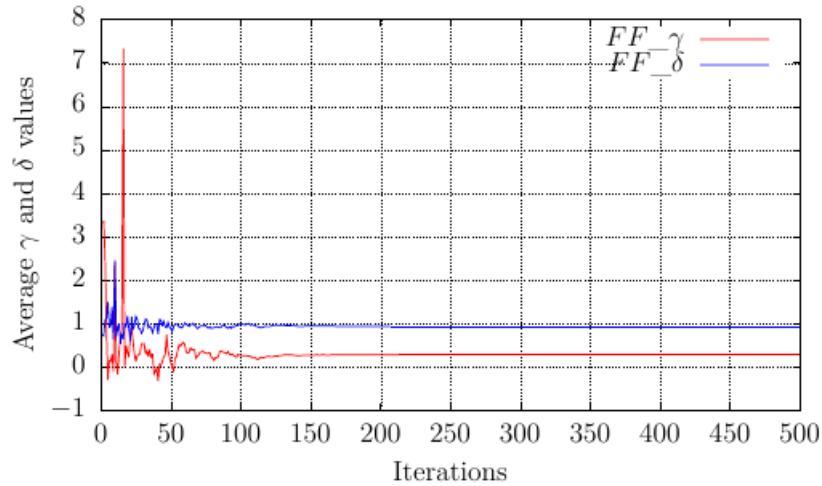


Figure 5: Variation of the average  $FF\_gamma$  and  $FF\_delta$  values over iterations for eil51 TSP instance

Figures 4 and 5 point out that the parameter values varies up to some number of iterations and then stabilize over an optimum value.



## 6 Concluding Remarks

The study has focused on implementing an ant colony algorithm to solve symmetric TSP problems whose parameters are handled by a self tuning firefly algorithm. The algorithm was successfully implemented and tested with standard TSP problems. According to the results obtained, some key conclusions can be drawn. In terms of optimization, the results show that the ACSFA performs well in finding the shortest path for a given TSP instance. The comparisons done with ACS and PSOACS shows ACSFA works well. Although the statistical analysis concludes that both ACSFA and PSOACS have same performance, ACSFA outperforms PSOACS by providing a parameter free environment. The self tuning framework worked fine with the firefly algorithm in tuning both parameters of ACS and FA. The graphical representations of the evolution of parameters of both ACS and FA clearly demonstrates the ability of the self tuning firefly algorithm. With these we can consider the new ACSFA as a better performer to solve TSPs using ACS.

For further development, this research encourages us to study the performance of the self tuning framework with other nature inspired algorithms such as particle swarm optimization, bees algorithm etc. Also since the increasing number of cities drops the performance of the algorithm, more experimentation should be done on the population size and the initialization of parameter ranges as well.

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