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KINETIC MODELS FOR GENERATION OF COSMOLOGICAL MATTER-ANTIMATTER ASYMMETRY AND BIOCHEMICAL RIGHT-LEFT ASYMMETRY

by

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INTRODUCTION

Richness and diversity of the universe depends on lack of perfect symmetry. The most symmetric universe would be a time reversible homogeneous gravitating system of photons. 'Physics' would become meaningless in such a situation as intelligent beings cannot evolve in this world, however, the physical laws as we know them today does not rule out this possibility. In fact if matter is particle—antiparticle symmetric a homogeneous photon universe[1] is the only possible realization.

The major microscopic asymmetries in the universe are (1) the arrow of time, i.e., the general irreversibility of all natural processes, (2) the inhomogeneity of space, i.e., the matter in the universe is lumped into galaxies placed far apart, (3) the chemical asymmetry, i.e., nearly symmetric forms of matter allowed by the microscopic laws does not exist with equal abundance, locally or globally.

Although, the microscopic world is largely asymmetric, the fundamental interactions among constituents of matter respects this attribute to a higher degree of accuracy. Arguments based on very general principles[2] have shown that the basic force are invariant with respect to combined operation of CPT (C = charge conjugation, P = parity, T = time reversal). Most processes involving elementary particles remain unchanged under individual operations C, P and T, yet the presence of small deviation from T and P invariance are also known. An interesting possibility is that the microscopic asymmetries of large scale universe are somehow connected with the microscopic ones of the fundamental interactions.

Thus far nobody has succeeded in detecting any relationship between violations in time reversal invariance and the arrow of time. Current developments in elementary particle physics and cosmology suggests that the inhomogeneity of space observed as the lumping of matter into galaxies may be related to elementary interactions at the very early stages of the universe. Magnetic monopoles or other massive particles could have triggered off condensation of galaxies. In such a case the inhomogeneity of the matter distribution is connected with 'quantization of mass'—another poorly understood asymmetry in the micro-world. Regarding chemical asymmetries there are more suggestive clues, the present thinking is that the matter-antimatter asymmetry of the universe is a result of the CP and baryon number violations at the bottom of fundamental interactions. A similar but not less important phenomenon is the biochemical right-left disparity seen in organic matter. The life on earth is based on L — amino acids and D sugars. Here againa- number of attempts have been made to relate this to parity violations in the weak interaction. In this work we present a mathematical model to illustrate that both their microscopic chemical asymmetries may have originated from similar kinetic mechanisms triggered by asymmetries in the basic forces.

Matter - Antimatter Asymmetry of the Universe

Several authors have proposed that the observed baryon asymmetry in the universe is a consequence of the baryon number and CP violating interactions occurring at very early stages of the universe[3-4]. The grand unified theories[5-6] of strong, weak and electromagnetic interactions predicts the existence of baryon number violating interactions which become significant at extremely high temperatures ($\sim 10^{30}$°K) reached during earliest times ($\sim 10^{-16}$S) after the big bang. In addition to baryon number and CP violating interactions, departure from thermal equilibrium is necessary to generate a net baryon excess.[4] Even if the time reversal invariance is violated, the CPT theorem guarantees that at the thermal equilibrium the number densities of baryons and antibaryons remain equal. The expansion of the universe is supposed to be the external agency that provides the necessary nonequilibrium conditions for generation of the baryon excess. Once the baryon violating interactions become insignificant due to the drop in temperature, the ratio of baryon to photon number remain constant or further decrease due to generation of additional entropy. The observed baryon to photon ratio in the universe[7] is $\sim 10^{-8} — 10^{-10}$. The estimates of this ratio depends on strengths of baryon number and CP violating interactions. Most estimates are highly sensitive to the CP violating parameters[3-4].

In general nonequilibrium processes are described by nonlinear rate equations and the simple estimates of the behaviour of such systems based on rates of individual reaction steps does not give meaningful results[8-9]. Since we have no knowledge of the details of all elementary particle reactions taking place at extremely high temperatures, where the baryon number violating processes are supposed to become important, it is impossible to write down the exact rate equations describing the cosmological generation of baryons. In this note we present a simple model which illustrates that in a nonequilibrium process, a large excess of baryons (or antibaryons) can be produced from an initially baryon-antibaryon symmetric state even if C and CP violations are arbitrarily small. It is shown that if baryons and antibaryons are generated in autocatalytic steps, the possibility exists that the rate equations describing the creation and annihilation of baryons and antibaryons exhibit a spontaneous breaking of the symmetry between baryons and antibaryons. A CP violation however small ensures that a large excess of one type of particles are produced.

In this model we assume that baryons and antibaryons are produced in autocatalytic steps. e g. reactions of the form

$$B + X \rightarrow Y + B + B \qquad (1)$$

$$\bar{B} + \bar{X} \rightarrow \bar{Y} + \bar{B} + \bar{B} \qquad (2)$$

where B denote a baryon, $\bar{B}$ an antibaryon and X, Y refer to two other species of elementary particles carrying zero baryon number. The presence of baryon number violations predicted by grand unified theories allows reactions of the form (1) and (2). For example X(Y) could be a meson and Y(X) a lepton. The SU(5) grand unified model[5] requires B-L (baryon number—lepton number) to remain constant in baryon number violating interactions. In such a case if X is a meson Y should be a lepton and if Y is a meson X should be an antilepton. For the purpose of the model we assume that X and $\bar{X}$ are present in excess so that their concentrations in the reacting medium remain practically constant. Baryons and antibaryons produced in reactions (1) and (2) undergo annihilations. If the annihilation products are dissipated fast from the reaction site, the annihilation reactions proceed in the forward direction. Also the removal of B and $\bar{B}$ due to annihilations and the presence of X and $\bar{X}$ in excess favours the proceeding of the reactions (1) and (2) in the forward direction. Of course an external agency is needed to maintain this nonequilibrium situation. Provided the above conditions are realized, the time variation of the baryon and antibaryon number densities are given by the rate equations,

$$\frac{dB}{dt} = KB - aB\bar{B} \qquad (3)$$

$$\frac{d\bar{B}}{dt} = \bar{K}\bar{B} - a\bar{B}B \qquad (4)$$

where K and $\bar{K}$ are constants.

To study the solutions of the coupled nonlinear equations (3) and (4), we first consider the case, where $K = \bar{K}$, corresponding to exact conservation of CP. When $K = \bar{K}$, it follows from (3) and (4) that

$$B - \bar{B} = Ae^{Kt}, \qquad (5)$$

where A = constant. If A = O, the baryon and antibaryon number densities remain equal at all times and setting $B = \bar{B}$ in (3) or (4) we obtain,

$$B(t) = \bar{B}(t) = \frac{K}{a\,(1 + be^{-Kt})}, \qquad (6)$$

where b is the integration constant. The expression (6) is the symmetric solution of the equations (3) and (4) with $K = \bar{K}$. It is seen that in the symmetric solution both B and $\bar{B}$ approach a steady state value $B = \bar{B} = K/a$ corresponding to $dB/dt = d\bar{B}/dt = O$. The system has no stable equilibrium position, later on it will be shown that the steady value $B = \bar{B} = K/a$ is an unstable stationary state.

The equations (3) and (4) with $K = \bar{K}$ also has two asymmetric solutions when $A \neq O$. If $A > O$ then $B > \bar{B}$ for all finite t. Eliminating $\bar{B}$ between (3) and (5) and solving the resulting equation in B by looking for solutions of the form $B = f(t)e^{Kt}$, we obtain for $A > O$, the following solution,

$$B = \frac{Ae^{Kt}}{1 - Ce^{\frac{-aA}{K} e^{Kt}}}$$

$$\bar{B} = \frac{Ae^{Kt} \quad \frac{aA}{CeK} e^{Kt}}{1 - Ce^{\frac{aA}{K} e^{Kt}}} \qquad (7)$$

where C ($O \leqslant C \leqslant 1$) is a constant. It is seen that in (7) $B \rightarrow \infty$, $\bar{B} \rightarrow O$ as $t \rightarrow \infty$ (Fig. 1). Changing A to —A and C to C^{-1} in (7) we obtain the other asymmetric solution,

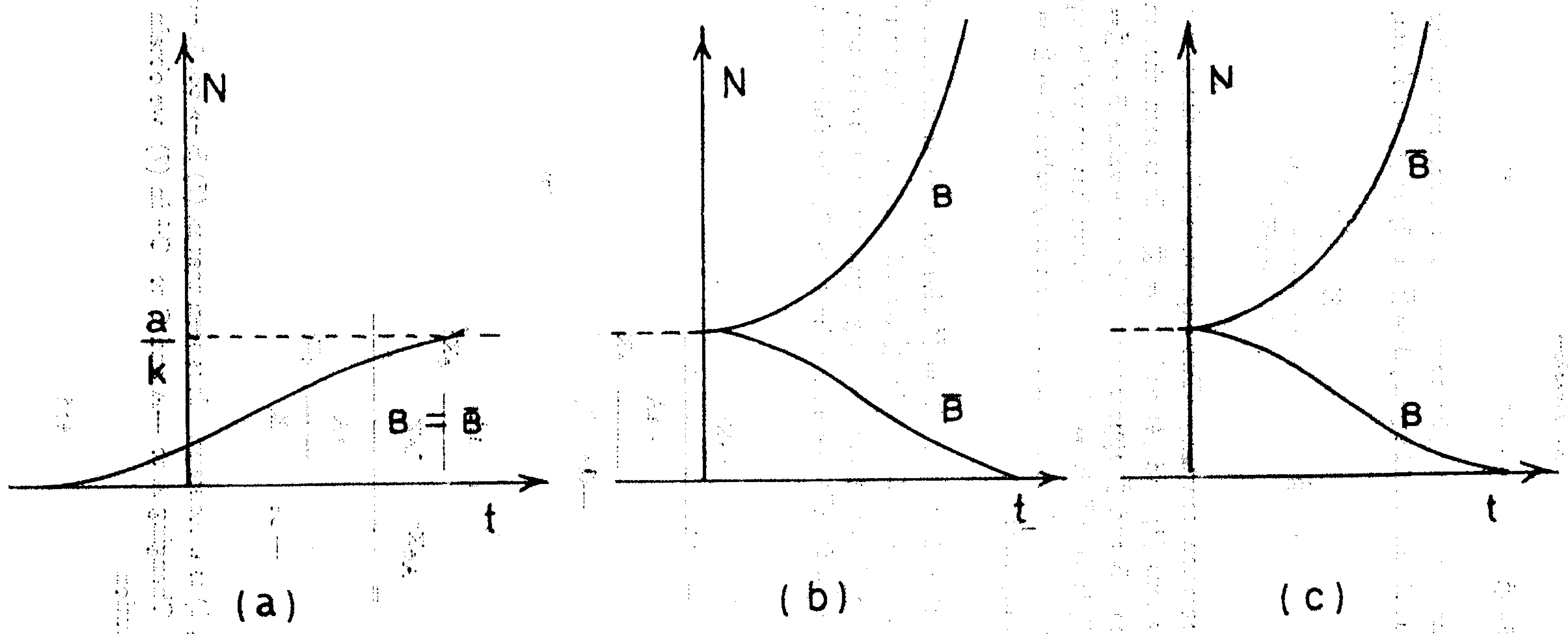

Fig. 1.

Fig. 1. The plot of N (B or $\bar{B}$) vs t (a) the symmetric solution where $B = \bar{B}$. (b) the asymmetric solution where $\bar{B} \to 0$, $B \to \infty$ as $t \to \infty$. (C) asymmetric solution where $B \to 0$, $\bar{B} \to \infty$ as $t \to \infty$.

$$\bar{B} = \frac{Ae^{Kt}}{1 - C^{-1} e^{-\frac{aA}{K} e^{Kt}}}$$

$$B = \frac{Ae^{Kt} C^{-1} e^{\frac{aA}{K} e^{Kt}}}{1 - C^{-1} e^{\frac{aA}{K} e^{Kt}}} \tag{8}$$

where the roles of B and $\bar{B}$ are interchanged, i.e. in the solution (8) $B \rightarrow O$, $\bar{B} \rightarrow \infty$ as $t \rightarrow \infty$ (Fig. 1). From equations (7) or (8) it follows that,

$$C = \left. \frac{\bar{B}}{B} \right|_{t \rightarrow -\infty}$$

Thus if we set $C = 1$ in (7) and (8) we obtain two asymmetric solutions of symmetric (symmetric with respect to B and $\bar{B}$) equations with a symmetric initial condition of having $B = \bar{B}$ at infinitely remote past. As mentioned earlier the symmetric solution (6) where $B(t) = \bar{B}(t) = K/a$ is unstable. To demonstrate the unstability of this solution we adopt the standard procedure of linearization used in stability analysis of systems described by nonlinear differential equations[10]. Linearizing (3) and (4) about the stationary point $B = \bar{B} = K/a$ by putting $B = K/a + b$, $\bar{B} = K/a + \bar{b}$, where b and $\bar{b}$ are the deviations of the baryon and antibaryon number densities from the steady values we obtain, the linearized equations,

$$\frac{db}{dt} = -K\bar{b}$$

$$\frac{d\bar{b}}{dt} = -Kb \tag{9}$$

whose solutions are,

$$b = Ce_1^{Kt} + C_2e^{-Kt}$$
$$\bar{b} = \bar{C}_1 e^{Kt} + \bar{C}_2 e^{-Kt} \qquad (10)$$

where C_1, C_2, $\bar{C}_1$, $\bar{C}_2$ are constants. The presence of the positive exponential factor of Kt ($K > O$) ensures that the symmetric solution is unstable[10]. Thus the solution of equations (3) and (4) with $K = \bar{K}$ exhibits spontaneous breaking of the symmetry between B and $\bar{B}$. The symmetric solution where $B(t) = \bar{B}(t)$ becomes unstable when it reaches the steady value and branches off into one of the asymmetric solutions where either B or $\bar{B}$ survives as $t \to \infty$ (Fig. 1.) In the absence of CP violation the above mechanism will not generate a net baryon number. The spontaneous breaking of the symmetry between B and $\bar{B}$ will yield excess baryon and antibaryon number densities with equal probabilities at different point so that there is no global violation of the baryon number. However, the interesting point is that the slightest CP violation, however small is sufficient to decide the direction of the spontaneous breaking of the symmetry between baryons and antibaryons. If $K - \bar{K} = \delta$ it is possible to show that the solution of (3) and (4) approaches (7) as $\delta \to +O$ and (8) as $\delta \to -O$ (Fig. 2). Hence in this model slightest CP violation is sufficient to ensure a rapid generation of baryon number from an initial state with zero baryon number.

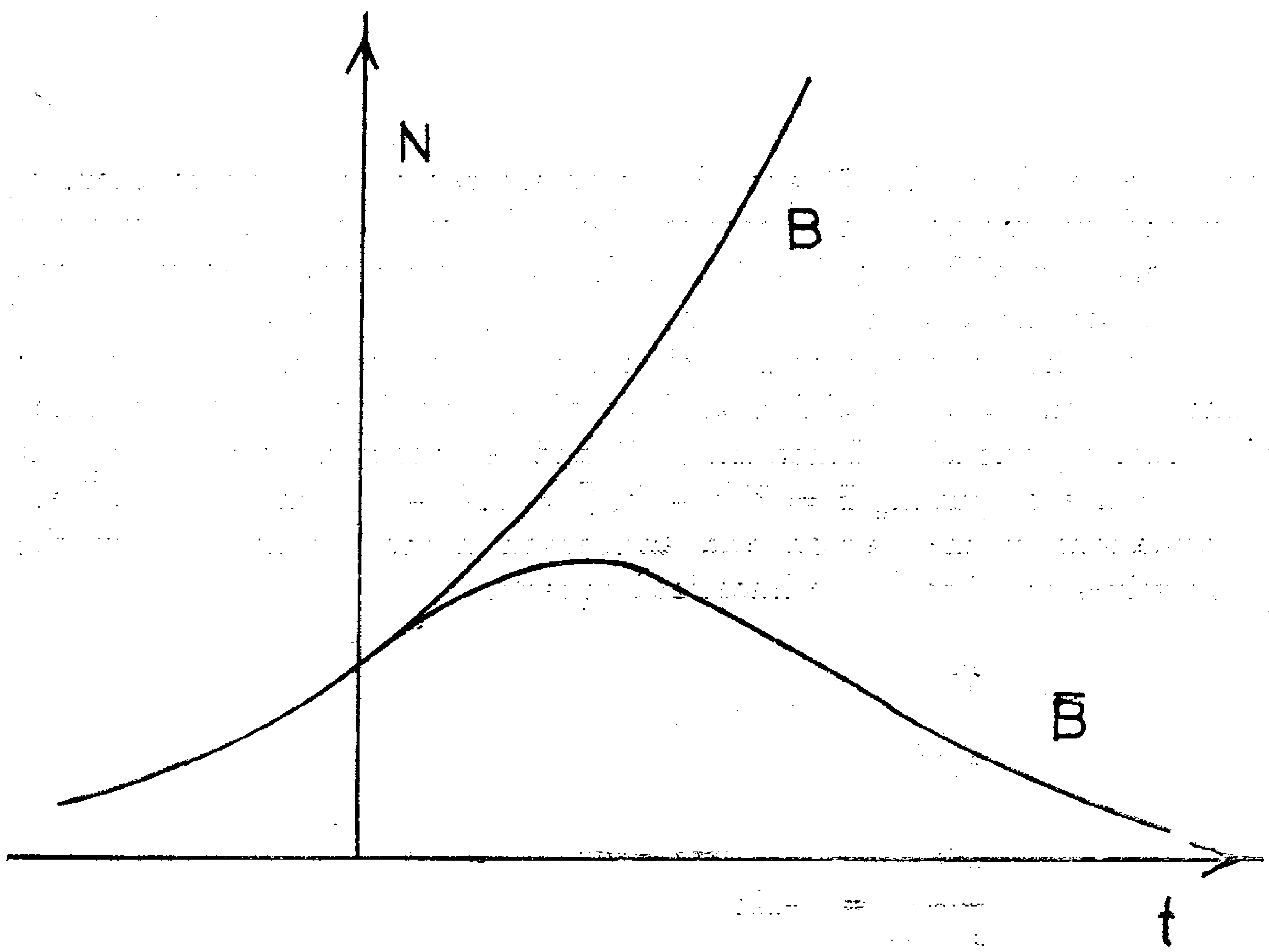

Fig. 2. The plot of N (B or $\bar{B}$) vs t when $k > K$ as $K - \bar{K} \to +0$ this solution develops in to the solution (b) of Fig. 1.

The model we have presented above is highly ideal, an external agency, the details of which we have left unspecfied, is needed to maintain the non-equilibrium. In cosmological generation of baryons the expansion of the universe is supposed to be the external agency that diverts the system from thermal equilibrium We do not claim that the nonequilibrium conditions needed for the model are realized in expansion of the universe at initial moments of birth. The model illustrates that although CP violation is necessary for generation of net baryon number, depending on reaction mechanisms the required CP violation could be arbitrarily small If the rates of generation of baryons and antibaryons in the autocatalytic reactions are greater than the expansion rate of the reacting medium and the rate of annihilation reactions are slower than the expansion rates, then the conditions needed for the above mechanism are realized at least for short durations of time [11]. Since the thresholds for the autocatalytic steps and the inverse annihilation reactions are different, the above constraints on the reaction rates in principle could be satisfied in a nonequilibrium process. There are also indications that grand unified particle interactions could not have thermalized the universe at very initial stages where the temperature is greater than the mass scale of baryon violating bosons[12]. The development of instabilities resulting spontaneous breaking of symmetries is a common property of nonequilibrium processes described by nonlinear rate equations[9-10]. Thus the actual mechanism of cosmological generation of baryons though very much more complicated could be similar to the mechanism we have proposed.

Biochemical L – D Asymmetry

It is generally believed that life orginated in the primeval oceans containing dissolved amino acids[13-15]. These acids are probably formed by the action of solar ultraviolet light or electrical discharges on the primitive atmosphere of methane, ammonia and water vapour[12-14]. The most significant event in the biochemical evolution would have been the first appearance of self-replicating molecules. To construct a model completely analogous to the earlier one we assume that the prebiotic medium was a racemic mixtures of amino acids and the stereoselection occurred after the development of self-replicating molecules. At the initial stages both L and D isomers of the primitive molecules would have existed in comparable concentrations. These molecules use amino acids of the correct type (L or D) as the food and regenerate units of the same type. However because of the difference in activation energies due to the presence neutral weak currents, the rate constants of replication K_L and K_D for the two types of molecules will not be exactly equal. Also the units of one type could sometimes interact with the other type resulting mutual inhibition of growth possibly owing to the formation of mixed polymers that cannot replicate. Mathematically, the rate equations describing the above processes are identical to (3) and (4) with $B = N_L$,

$\bar{B} = N_D$, $K_L = K$, $K_D = \bar{K}$ where N_L and N_D are the concentrations of the two types of molecules. Again it is at once seen that depending on which rate constant is greater only one of the enantiomers can survive.

Admittedly these models over-simplifies the reaction kinetics, the actual rate equations would be far more complicated. Nevertheless even more involved nonlinear symmetric rate equations are expected to exhibit spontaneous breaking of the symmetry and any difference between rate constants however small guarantees global breaking of the symmetry.

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