Vidyodaya J. Arts, Sci., Lett., Silver Jubilee, pp. 126-129

ONE-PARAMETER VERSUS TWO-PARAMETER MODEL LIFE—TABLE SYSTEMS

by

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A set of Model Life Tables presents the mortality experiences according to age and sex to be expected in human populations under a range of mortality conditions. Such tables have been used extensively for the graduation, adjustment and extension of the limited and defective demographic data from many developing countries. For a particular application, a table can be selected by using some clues about or recorded aspect of the mortality of the population in question.

All the life table functions in a model table can be derived if we know a complete set of q_x values for all ages, where q_x is the probability that a person who has survived to age x will die before age $x+1$. Apart from the early years of life it is usually sufficient to obtain values of ${}_5q_x$, the probability of dying within next 5 years for a person now aged x. By analysing past records, demographers have obtained plausible upper and lower limits for the values of q_x at each age x. It has also been noticed that if a country has a value of q_x near the lower limit at one age, it is likely to be near this limit at other ages also, and it is extremely improbable that at any age it is near the upper limit. One reason for this may be the broad similarity of many of the factors affecting mortality at different ages. Thus, mortality rates at all ages can be expected to be low in a country where the standard of living is high, environmental sanitation is good, and where medical services incorporating modern knowledge are proficient and extensive.

In such a country deaths due to under-nourishment and infectious diseases can be expected to be low, but deaths due to degenerative causes may be important. The result to be expected is moderate mortality in infancy, very low mortality in childhood and early adult ages, and rather higher death rates at older ages, when the degenerative causes become more prominent. In a country without these favourable conditions, infectious disease and under-nourishment cause many deaths at all ages, in addition to deaths due to degenerative causes. The prevalence of these conditions is different in different populations. Thus, there is a typical pattern of death rates by age when mortality is high, moderate or low. Death rates for one age group are highly correlated with death rates for other age groups, so it is possible to represent a full schedule of mortality to a tolerable approximation by the suitable choice of a small number of parameters. Model life tables are an expression of this inter-relationship.

A large number of alternative model life tables have been published. The most widely applied are those of United Nations (1955), Coale and Demeny (1966) and Brass (1971).

One-parameter Model Life—Table Systems

These tables have been derived by assuming that differences in the mortality experiences of different populations can be satisfactorily explained by variation in the value of one parameter. The U.N. and the Coale and Demeny Model Life Tables are of this type; although the latter achieves greater flexibility through four separate one-parameter families.

The U.N. model life tables were based on a large number of life tables from countries with highly accurate data. A study was made of the regressions of ${}_4q_1$ on q_o ${}_5q_5$ on ${}_4q_1$ and ${}_5q_{x+5}$ on ${}_5q_x$ for $x = 5, 10, 15,$ .... A set of model life tables was computed by fitting quadratic regressions between the q_x values of adjacent age groups starting from a specified value of q_o. The U.N. (1956) published 24 model tables with values of q_o chosen so that the expectation of life at birth, $e^o{}_x$, took suitable values in the range 20 to 73.9 years.

Further examination of recorded life tables has shown that the U.N. model tables do not always produce a good fit to the diverse observed patterns of mortality. Coale and Demeny (1966) divided a large number of recorded life tables into 4 families, namely "North," "East" "South" and "West," in such a way that the mortality patterns within each family were as uniform as possible.

Within each family a very high correlation (usually above 0.95) between mortality rates at different ages was found and these correlations were much higher than the correlations found when all four families were pooled together. Thus Coale and Demeny constructed separate one-parameter model life tables for each of the four families by regressing q_o values of adjacent age groups.

Although the Coale and Demeny model life tables are more flexible than the U.N. life tables, it cannot be assumed that the mortality experience of every population can be approximated by one of the four families. Three of the families were derived wholly from European life tables and even in the fourth family a large number of European life tables were included. The mortality experience of European nations may not be applicable to the situation of developing countries in the process of mortality change as well as in social and cultural characteristics. Analysing the mortality experience of some developing countries where more reliable data are available Adlakhan

(1972) argues that mortality patterns in developing countries may deviate substantially from any of the existing one-parameter life table models. Brass (1972) comes to similar conclusions from a study of several other countries.

Brass's Two-Parameter Logistic Model Life-Table System

The Brass method of constructing a model life table is to take the logistic transformation of the survivorship function in a life table chosen as a standard, and then to consider the life tables generated by linear transformations of the standard logits. Different standards may be used depending on the area of application. For a given application a standard may be obtained by averaging a number of observed life tables. Thus, if l_{sx} is the survivorship in some arbitary member of the system, then.

$$Y_x = A + BY_{sx}$$

$$\text{Where } Y_x = \tfrac{1}{2} \text{Log}_e \frac{(1-l_x)}{l_x}; \qquad Y_{sx} = \tfrac{1}{2} \text{Log}_e \frac{(1-l_{sx})}{l_{xs}}$$

and A and B are disposable parameters.

The logit Y_{sx} is zero when l_{sx} is 0 5, i.e. at the age to which half the births of the standard population survive. For the standard used by Brass (1971) this age is 51 years. Since Y_x is then equal to A this parameter can be regarded, in a sense, as measuring the level of mortality. A positive value of A gives an l (x) survivorship of less than 0.5 at age 51 years. If B is equal to unity, Y_x–Y_{sx} has the value A at all ages, and consequently, the difference between l_x and l_{sx} are in the same direction throughout. The life table defined by l_x then has a heavier mortality than the standard. If B is greater than one, however, the difference Y_x—Y_{sx} will be less than A at ages below 51 years and greater than A at ages over 51. Thus, the life survivorship ratios l(x) fall off more rapidly up to age 51. than for a population with the same A, but with B equal to one, and less rapidly after age 51. The opposite holds for B less than one. It is convenient, therefore, to call B the "slope" of mortality (Brass 1971).

There is no need to calculate and publish a selection of life tables from the system since, as need arises, a suitable life table can be generated by computer. A set of standard life table logits, mainly suitable for African populations, has been published, (Carrier and Hobcraft, 1971).

The Brass system is more satisfactory than the U.N., or the Coale and Demeny model-life lables for exhibiting the variety of age patterns found in observed life tables. According to Carrier and Hobcraft (1971) the trend of mortality can be fit better by using Brass' system than the U.N. one parameter system if the country is one that has only recently started a mortality decline.

Since this is the main feature in many of the developing countries where model life tables are intended to be applied, the Brass two-parameter system should be used in preference to U.N., or Coale and Demeny Systems, very recently, Zaba (1979) has extended these ideas to a four-parameter logit life table system.

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