Performance Optimized Expectation Conditional Maximization Algorithms for Nonhomogeneous Poisson Process Software Reliability Models

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***Abstract*—Nonhomogeneous Poisson process (NHPP) and software reliability growth models (SRGM) are a popular approach to estimate useful metrics such as the number of faults remaining, failure rate, and reliability, which is defined as the probability of failure free operation in a specified environment for a specified period of time. We propose performance-optimized expectation conditional maximization (ECM) algorithms for NHPP SRGM. In contrast to the expectation maximization (EM) algorithm, the ECM algorithm reduces the maximum-likelihood estimation process to multiple simpler conditional maximization (CM)-steps. The advantage of these CM-steps is that they only need to consider one variable at a time, enabling implicit solutions to update rules when a closed form equation is not available for a model parameter. We compare the performance of our ECM algorithms to previous EM and ECM algorithms on many datasets from the research literature. Our results indicate that our ECM algorithms achieve two orders of magnitude speed up over the EM and ECM algorithms of [1] when their experimental methodology is considered and three orders of magnitude when knowledge of the maximum-likelihood estimation is removed, whereas our approach is as much as 60 times faster than the EM algorithms of [2]. We subsequently propose a two-stage algorithm to further accelerate performance.**

***Index Terms*—Expectation conditional maximization (ECM) algorithm, nonhomogeneous Poisson process (NHPP), software reliability, software reliability growth model, two-stage algorithm.**

NOMENCLATURE

Acronyms

EM	Expectation-maximization.
ECM	Expectation conditional maximization.
DSS	Delayed S-shaped model.
CDF	Cumulative distribution function.
LL	Log-likelihood function.
RLL	Reduced log-likelihood function.
MLE	Maximum-likelihood estimation.
MVF	Mean value function.
NHPP	Nonhomogeneous Poisson process.
SRGM	Software reliability growth models.
GO	Goel–Okumoto model.
ISS	Inflexion S-shaped model.

Notation

$m(t)$	MVF of NHPP.
$\lambda(t)$	Instantaneous failure rate.
$F(t)$	Cumulative distribution function of software fault detection process.
a	Number of latent faults at start of testing.
b	Scale parameter of Weibull SRGM testing.
c	Shape parameter of Weibull SRGM testing.
ϕ	Rate at which bathtub transitions. from second to final phase.
Ψ	Inflexion parameter.
r	Inflexion rate.
Γ	Gamma function.
F	Polygamma function.
G	MeijerG function.
N	Total number of faults.
n	Observed number of faults.
m	Unobserved number of faults.
$\mathbf{T}$	Vector of failure times.
t_i	Time of the ith failure.
t_n	nth observed failure.
t_{obs}	Time at which testing stopped.
s	Mission time.
x	Observed data.
z	Unobserved data.
Θ	Vector of model parameters.
p	Number of model parameters.
Δ	Vector of conditional maximization steps.
δ	Subvector of conditional maximization steps.
$\Theta_i^{(j)}$	ith model parameter in the jth iteration

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