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n −Fold m −Valued Logic

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Abstract

In this paper, the basic concepts of two valued logic, many valued logic and catuskoti logic are discussed. We comprehensively analyzed the statement in different branches of logic. We define a 'fold' and a 'statement' in catuskoti. We further defined a 'fold' in general, a 'multi valued statement' and 'n −fold $m-$ valued logic' (where n, m are integers greater than or equal to two). We identify that every branch of logic could be expressed in the form of n −fold $m-$ valued logic. We explain the problem of drawing a tangent to a curve and Zeon's arrow paradox by using the 4 −fold $2-$ valued logic. Truth tables for negation, conjunction, disjunction and implication in the 4 −fold $2-$ valued logic are discussed in this paper.

Keywords: Catuskoti; Many valued logic; Multi valued statement; n −fold m − valued logic.

1. Introduction

Aristotle introduced the classical two valued logic that is called Aristotelian logic. So far, it has been dominating the world. A statement in classical two valued logic is defined as a sentence that can be considered as true or false. However, there are sentences with number of truth values greater than two. Using the sentences that have truth values other than true and false, many valued logics are defined. Such logics are three valued logic, four valued logic, k −valued logic (where k is an integer greater or equal to two) and infinite valued logic.

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In Buddhist philosophy, there is another different type of logic called catuskoti or tetra lemma or four corner logic or four-fold logic. In early Buddhist logic, it was the standard to assume that for any state of affairs, there were four possibilities; that it held, that it did not, both or neither [1]. Within the literature survey, we could not find clear definitions of a statement and a fold in Catuskoti. In this paper, we explain some phenomenon, which cannot be understood in classical two valued logic by using 4-fold 2-valued logic. This work is limited to a discussion of the basic concepts of logics up to the operations of statements in $n-$fold $m-$valued logic. The axiomatizations of logics are not discussed in this paper and the applications are confined to the $4-$fold $2-$valued logic.

2. Logic

The logics were developed in many cultures, including Judeo Christian, Catholic, Hindu, Islamic and Buddhist. A logic is a method of obtaining conclusion from a given set of statements in a system of knowledge. Logic is discussed mainly in categories, namely, Aristotelian logic, many valued logic, fuzzy logic and Catuskoti logic. Scientists, Mathematicians and Philosophers have been using Aristotelian logic to infer the conclusions in systems of knowledge. The logic in Catuskoti, which is a Buddhist epistemological concept, is also applied in systems of knowledge in cultures based on Buddhist Philosophy. A logic is a set of concepts, and a set of rules involving the concepts, which is used for arguments by human beings. The set of concepts and the set of rules are relative to the culture of the human beings.

3. Preliminary Concepts

The following definitions of concepts are from various authors as stated in the literature.

3.1 Two Valued Statement

A sentence that is capable of being either true or false is defined as a statement or a proposition [2]. It is also called declarative sentence [2]. Any statement is either true or false but not both.

3.2 Aristotelian Logic

Aristotle identified that every science could be described as a system starting from a finite set of basic rules called axioms and undefined terms to derive results as well as in Euclidean Geometry. The basic principles to derive the results from the axioms were the same for every science. Aristotelian logic is sometimes defined as the science of reasoning [3].

3.2.1 Concepts and Fundamental Principles of Aristotelian Logic

In the following sections the concepts and fundamental principles of Aristotelian logic are discussed.

3.2.1.1 Subject

A subject is typically an individual entity, for example a dog, a table, a woman or a group such as a set of numbers, a group of people etc [3].

3.2.1.2 Predicate

A predicate is the property or attribute or mode of existence of the subject [3]. A given subject possesses or does not possess a given predicate. The color (property) of the box (subject) is red. All the women (subject) in the world are kind (predicate).

3.2.2 Fundamental Principles of Predication in Aristotelian Logic

In the literature, the fundamental principles of predication are stated as follows.

3.2.2.1 The Law of 'Identity

Everything is what it is. In symbolic form, it can be expressed as A is A [3]. This implies that an object called A exists. Modern formulation of the above law is $\forall x\big(A(x) \Rightarrow A(x)\big)$ [3].

3.2.2.2 The Law of 'Non-Contradiction'

It is impossible that for a subject both to be and not to be. A given predicate cannot both belong and not belong to a given subject in a given respect at a given time [3]. This implies that contradictions do not exist. That is A and $\neg A$ cannot be at the same time. Modern formulation of the above law is $\neg\exists x\big(A(x) \wedge \neg A(x)\big)$.

3.2.2.3 The Law of 'Either-Or'

Everything must either be or not be. A given predicate either belongs or does not belong to a given subject in given respect at a given time [3]. That is either Aor $\neg A$ be at the same time. This law is also called The Law of Excluded Middle [3]. Modern formulation of the above law is $\forall x\big(A(x) \vee \neg A(x)\big)$ [3].

3.3 Many Valued Logics

So far we have been confined to statements (propositions) which are either true or false. We call these statements two valued statements, and a truth value was assigned to a statement depending on whether it was true or false. However, there may be sentences with more than two truth values. We call these sentences $n-$valued statements (propositions) defined on the number of truth values n that the sentence may take. A logic that has many valued statements are called many valued logics. Many valued logic is also known as multi-valued logic or multiple-valued logic.

3.3.1 Three Valued Logics

An $n-$ valued logic with three truth values is called three valued logic. The three values may be defined as true, false and unknown in western logic. These three truth values are denoted by T, F and I. Truth value of a $3-$valued statement $\mathcal{A}$ is denoted by $T(\mathcal{A})$ [4]. $max\{T,F\} = T$, $max\{T,I\} = T$, $max\{I,F\} = I$, $min\{T,F\} = F$, $min\{T,I\} = I$ and $min\{I,F\} = F$. Let $\mathcal{A}$ and $\mathcal{B}$ be two $3-$ valued statements in a $3-$valued logic. In the definition of truth values of conjunction and disjunction are defined by using formulae

$T(\mathcal{A} \wedge \mathcal{B}) = max\{T(\mathcal{A}), T(\mathcal{B})\}$ and $T(\mathcal{A} \vee \mathcal{B}) = min\{T(\mathcal{A}), T(\mathcal{B})\}$ respectively [4].

3.3.1.1 Kleene Logic (strong)K_3 and Priest Logic P_3

In the literature, the difference between Kleene logic (strong) $\boldsymbol{K_3}$ and Priest logic $\boldsymbol{P_3}$ was stated as follows. In Kleene's logic $\boldsymbol{K_3}$ can be interpreted as being "underdetermined", being neither true nor false, while in Priest's logic $\boldsymbol{P_3}$ can be interpreted as being "overdetermined", being both true and false. Negation, conjunction, disjunction and material implication in Kleene (strong) logic $\boldsymbol{K_3}$ and Priest logic $\boldsymbol{P_3}$ are defined as follows [5]. In defining the truth values of $\mathcal{A} \Rightarrow_K \mathcal{B}$ and $\mathcal{A} \Leftrightarrow_K \mathcal{B}$, it is used that the equivalence $\mathcal{A} \Rightarrow_K \mathcal{B} \equiv \neg\mathcal{A} \vee \mathcal{B}$.

Table 1: Truth Values of Negation in K_3 and P_3 [5]

$\mathcal{A}$	$\neg\mathcal{A}$
T	F
F	T
I	I

Table 2: Truth Values of Other Operators of K_3 and P_3 [5]

$\mathcal{A}$	$\mathcal{B}$	$\mathcal{A} \wedge \mathcal{B}$	$\mathcal{A} \vee \mathcal{B}$	$\mathcal{A} \Rightarrow_K \mathcal{B}$	$\mathcal{A} \Leftrightarrow_K \mathcal{B}$
T	T	T	T	T	T
T	I	I	T	I	I
T	F	F	T	F	F
I	T	I	T	T	I
I	I	I	I	I	I
I	F	F	I	I	I
F	T	F	T	T	F
F	I	F	I	T	I
F	F	F	F	T	T

3.3.1.2 Bochvar's Internal Three-Valued Logic

Another three-valued logic is Bochvar's "internal". It is denoted by B_3^I. It is also called Kleene's weak three-valued logic. Truth tables for negation and bi-conditional in Bochvar's internal three-valued logic are the same as in Kleene logic (strong) K_3 and Priest logic P_3. Material implication in Bochvar's internal three-valued logic is defined by using the equivalence $\mathcal{A} \Rightarrow \mathcal{B} \equiv \neg\mathcal{A} \vee \mathcal{B}$ [4]. Truth tables for conjunction, disjunction and material implication in Bochvar's internal three-valued logic are as follows.

Table 3: Truth Values of Other Operators in Bochvar's Internal Three-Valued Logic [4]

$\mathcal{A}$	$\mathcal{B}$	$\mathcal{A} \wedge_+ \mathcal{B}$	$\mathcal{A} \vee_+ \mathcal{B}$	$\mathcal{A} \Rightarrow_+ \mathcal{B}$	$\mathcal{A} \Leftrightarrow_+ \mathcal{B}$
T	T	T	T	T	T
T	I	I	I	I	I
T	F	F	T	F	F
I	T	I	I	I	I
I	I	I	I	I	I
I	F	I	I	I	I
F	T	F	T	T	F
F	I	I	I	I	I
F	F	F	F	T	T

3.3.1.3 Lukasiewicz Logic

The Lukasiewicz logic has the same tables for conjunction, disjunction, and negation as the Kleene logic given above since the truth values are defined by the following formulae $T(\mathcal{A} \wedge \mathcal{B}) = min\{T(\mathcal{A}), T(\mathcal{B})\}$ and $T(\mathcal{A} \vee \mathcal{B}) = max\{T(\mathcal{A}), T(\mathcal{B})\}$ respectively. However, it differs in its definition of material implication. This section follows the presentation from Malinowski's chapter of the handbook of the history of logic. $\mathcal{A} \Rightarrow \mathcal{B}$ is denoted by $IMP_L(\mathcal{A}, \mathcal{B})$.

Table 4: Truth Values of $IMP_L(\mathcal{A}, \mathcal{B})$ [4].

$\mathcal{A}$	$\mathcal{B}$	$\mathcal{A} \Rightarrow \mathcal{B}$
T	T	T
T	F	F
T	I	I
F	T	T
F	F	T
F	I	T
I	T	T
I	F	I
I	I	T

3.3.2 Four Valued Logic

A logic where statements are assigned four truth values namely true, false, over determined and under determined denoted by T, F, B and D is defined as a four-valued logic [4].

3.3.2.1 Belnap Logic

Belnap's logic is denoted by B_4 combines K_3 and P_3. Negation, conjunction and disjunction in Belnap's logic $B_4$are defined as follows. Belnap logic has no table for an implication connective [4].

Table 5: Truth Values Negation in B_4 [4].

$\mathcal{A}$	$\neg\mathcal{A}$
T	F
B	B
N	N
F	T

Table 6: Truth Values of Other Operators in B_4 [4].

$\mathcal{A}$	$\mathcal{B}$	$\mathcal{A} \wedge_+ \mathcal{B}$	$\mathcal{A} \vee_+ \mathcal{B}$
T	T	T	T
T	B	B	T
T	N	N	T
T	F	F	T
B	T	B	T
B	B	B	B
B	N	F	F
B	F	F	B
N	T	N	T
N	B	F	T
N	N	N	N
N	F	N	F
F	T	F	T
F	T	F	F
F	N	F	N
F	F	F	F

3.3.3 Gödel's Many Valued Logics

In 1932, Gödel defined a family of many-valued logics denoted by G_k with finitely many truth values,

$$0, \frac{1}{k-1}, \frac{2}{k-1}, \dots\dots\dots \frac{k-2}{k-1}, 1.$$

For example, G_3 has the truth values

$$0, \frac{1}{2} \text{ and } 1.$$

and G_4 has

$$0, \frac{1}{3}, \frac{2}{3} \text{ and } 1.$$

In a similar manner, he defined a logic with infinitely many truth values, G_∞ in which the truth values are all the real numbers in the interval $[0, 1]$ [4]. The truth values of a statement $\mathcal{A}$ in Gödel logics G_k is denoted by $T(\mathcal{A})$.

3.3.3.1 Truth Values of Conjunction in G_k

Let $\mathcal{A}$ and $\mathcal{B}$ be any two statements. The truth value of a conjunctive statement is defined as follows $T(\mathcal{A} \wedge_{G_k} \mathcal{B}) = \min\{T(\mathcal{A}), T(\mathcal{B})\}$, $T(\mathcal{A} \wedge_{G_\infty} \mathcal{B}) = \lim_{k\to\infty}(\min\{T(\mathcal{A}), T(\mathcal{B})\})$ [4].

3.3.3.2 Truth Values of Disjunction in G_k

Let $\mathcal{A}$ and $\mathcal{B}$be any two statements. The truth value of a disjunctive statement is defined as follows

$T(\mathcal{A} \vee_{G_k} \mathcal{B}) = \max\{T(\mathcal{A}), T(\mathcal{B})\}$, $T(\mathcal{A} \vee_{G_\infty} \mathcal{B}) = \lim_{k\to\infty}(\max\{T(\mathcal{A}), T(\mathcal{B})\})$ [4].

3.3.3.3 Negation in G_k

Let $\mathcal{A}$ be any statement. The truth value of negation of statement $\mathcal{A}$ is defined as follows:

$$T(\neg_{G_k}\mathcal{A}) = \begin{cases} 1 \text{ if } T(\mathcal{A}) = 1 \\ 0 \text{ if } T(\mathcal{A}) = 0 \end{cases} \text{ [4].}$$

3.3.3.4 Implication in G_k

Let $\mathcal{A}$ and $\mathcal{B}$ be any two statements. The truth value of implication is defined as follows:

$$T(\mathcal{A} \Rightarrow_{G_k} \mathcal{B}) = \begin{cases} 1 \text{ if } T(\mathcal{A}) \leq T(\mathcal{B}) \\ 0 \text{ if } T(\mathcal{A}) > T(\mathcal{B}) \end{cases} \text{ [4].}$$

3.4 Fuzzy Logic

A logic with statements considering two values namely true and false, and assigning two values p and $1-p$ for truth and false respectively, where $0<p<1$ is defined as Fuzzy logic. There are many infinitely many Fuzzy logics, where each Fuzzy logic is a two-valued logic, in which truth value of any statement is a real valuebetween zero and one.

3.5 Catuskoti

Catuskoti known as tetra lemma in Greek is a peculiar reasoning method that is used to explain some Buddhist thoughts by Lord Buddha. Even today, Buddhist disciples in Asian countries apply Catuskoti to explain the uncertainty of the human life. Catuskoti is a Pali ward. Catus represents four and the meaning of the koti is conner or fold.

3.5.1 Statement in Catuskoti

We may define a statement in Catuskoti as follows. Let $\mathcal{A}$ be a two-valued statement and $\neg\mathcal{A}$ be the negation of $\mathcal{A}$. If $\mathcal{A}$ and $\neg\mathcal{A}$ can be assigned the same truth value true or false at the same time, then $\mathcal{A}$ is defined as a statement in Catuskoti.

3.5.2 Fold in Catuskoti

In Catuskoti, the truth values of a statement $\mathcal{A}$ and its negation $\neg\mathcal{A}$ can be arranged as the following four different ways.

(a) $\mathcal{A}$ is true and $\neg\mathcal{A}$ is false.
(b) $\mathcal{A}$ is false and $\neg\mathcal{A}$ is true.
(c) $\mathcal{A}$ is true and $\neg\mathcal{A}$ is true.
(d) $\mathcal{A}$ is false and $\neg\mathcal{A}$ is false.

We may define a corner or a fold in Catuskoti as follows. Any one of the above ways is defined as a corner or a fold in Catuskoti. Since there are four different ways regarding any statement $\mathcal{A}$ and its negation $\neg\mathcal{A}$ it is said to be Catuskoti. The first two ways correspond to Aristotelian logic and the last two ways are not sensory perceptible.

3.5.3 Logic in Catuskoti

A logic having two valued statements and including the above four ways is defined as a Catuskoti logic.

4. Logics with Many Folds

Let $\mathcal{A}$ be a statement in any logical system and its negation be $\neg\mathcal{A}$. Two valued logic and many valued logics, which we have considered so far do not include the cases where $\mathcal{A}$ and $\neg\mathcal{A}$ are both true at the same time, or $\mathcal{A}$

and $\neg\mathcal{A}$ are both false at the same time. There are no such cases in Aristotelian logic. In this section, we are concerned with logics where $\mathcal{A}$ and $\neg\mathcal{A}$ are both true, or $\mathcal{A}$ and $\neg\mathcal{A}$ are both false at the same time.

4.1 Multi Valued Statement

We may define multi valued statement as fallows. Let A be two valued or a many valued statement and $\neg A$ be the negation of A. If A and $\neg A$ can be assigned the same truth value true or false at the same time then, A is defined as a multi valued statement.

4.2 Fold in General

Let $\mathcal{A}$ be a statement in any logic and $\neg\mathcal{A}$ be its negation. We define an assignment of possible truth values to the statements $\mathcal{A}$ and $\neg\mathcal{A}$ as a fold. The maximum number of folds in an m −valued logic is m^2. Example, in Aristotelian logic there are two truth values so the maximum number of fold is 2^2. They are given by the following table.

Table 7: Folds of Aristotelian Logic

$\mathcal{A}$	$\neg\mathcal{A}$
T	F
F	T
T	T
F	F

Example, in $3 -$ valued logic, there exists three truth values for any statement $\mathcal{A}$ so the maximum number of folds $3 -$ logic is $3^2 = 9$.

Table 8: Folds for a Three-Valued Logic

$\mathcal{A}$	$\neg\mathcal{A}$
T	T
T	F
T	I
F	T
F	F
F	I
I	T
I	F
I	I

4.4 n −Fold m −Valued Logic

A logic having an n −number of folds, and an m −number of truth values is defined as an n −fold m −valued logic. Aristotelian logic is a 2 −fold 2 −valued logic. Gödel's $3-$ valued logic (G_3) is a 3 −fold 3 −valued logic.

4.4.1 Logic in Catuskoti is a 4 −Fold 2 −Valued Logic

Let $\mathcal{A}$ be statement in a logic and $\neg A$ be its negation. Then the logic is called a 2 −fold 2 −valued logic if the possible assignments of truth values of A and $\neg A$ are as follows. There are only two truth values $T$and F as in the case of Aristotelian logic, but they can be assigned in four ways. Hence, it is a 4 −fold 2 −valued logic. The truth values of the conjunctive statement $\mathcal{A} \wedge \mathcal{B}$ and the disjunctive statement $\mathcal{A} \vee \mathcal{B}$ are defined by

$T(\mathcal{A} \wedge \mathcal{B}) = min\{T(\mathcal{A}), T(\mathcal{B})\}$ and $T(\mathcal{A} \vee \mathcal{B}) = max\{T(\mathcal{A}), T(\mathcal{B})\}$ respectively.The material implication in 4 −fold 2 −valued logic is defined by using the equivalence $\mathcal{A} \Rightarrow \mathcal{B} \equiv \neg\mathcal{A} \vee \mathcal{B}$

Table 9: Truth Values for 4- Fold 2- Valued Logic

$\mathcal{A}$	$\neg\mathcal{A}$	$\mathcal{B}$	$\mathcal{A} \wedge \mathcal{B}$	$\mathcal{A} \vee \mathcal{B}$	$\neg\mathcal{A} \vee \mathcal{B}$
T	F	T	F	T	F
F	T	F	F	T	T
T	T	T	T	T	T
F	F	F	F	F	F
T	F	F	F	T	F
F	T	T	F	T	T
T	T	F	F	T	T
F	F	T	F	F	T

Taking $\mathcal{B}$ to be $\neg\mathcal{A}$ we obtain the following table.

Table 10: Truth Table in Catuskoti Related to $\mathcal{A}$ and $\neg\mathcal{A}$

$\mathcal{A}$	$\neg\mathcal{A}$	$\mathcal{A} \wedge \neg\mathcal{A}$	$\mathcal{A} \vee \neg\mathcal{A}$
T	F	F	T
F	T	F	T
T	T	T	T
F	F	F	F

From the above table, it is found that the truth value of $\mathcal{A} \wedge \neg\mathcal{A}$ is true in 4 −fold 2 −valued logic whereas it is false in Aristotelian logic which is a 2 −fold 2 −valued logic. $\mathcal{A} \wedge \neg\mathcal{A}$ is not a contradiction in 4 −fold 2 −valued logic.

4.4.2 Applications of 4 −Fold 2 −Valued Logic in Mathematics

In the following sections, some of the applications of 4 −fold 2 −valued logic are discussed.

4.4.2.1 Tangents to a Curve

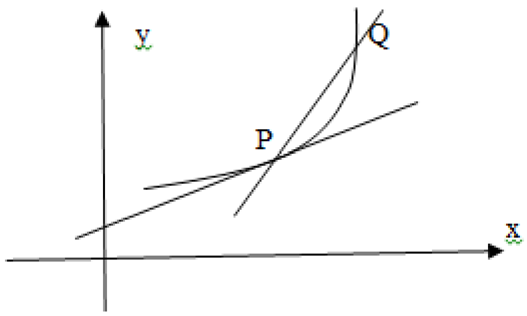

Figure 1: Tangent to a Curve

Let P and Q be two points on a given curve, where P is a fixed point. The chord PQ and the arc PQ are different if P and Q are distinct points. In order for the chord PQ and arc PQ exist P and Q have to be distinct points on the curve. Now consider the case when Q approaches gradually to P through the curve. Even if Q is very close to P as long as they remain distinct there are two different geometrical objects, one being the chord PQ and the other being the arc PQ. It is said that in the limit as Q tends to P the chord PQ becomes the tangent at P, and the arc PQ also in the limit becomes the tangent at P. However, the question arises whether Q coincides with P. If Q coincides with P, then there is no chord or arc and hence, we cannot obtain the tangent by considering the case where Q coincides with P. In order to obtain a straight line, we have to have two distinct points. In other wards P and Q must be distinct points. However close, they may be. On the other hand, we draw a tangent to the curve at the point P and not the points Q or neighboring points of P on the curve. Hence, if we consider the tangent as the limiting case of chord PQ or arc PQ, then it is necessary that Q has to be distinct from P as well as to be coincident with P. Thus, we can say that P and Q are distinct points as well as coincident points (not distinct) have to be true. Hence, we cannot draw tangents to a curve in Aristotelian Logic which is 2 −fold 2 −valued Logic where the points P and Q are either distinct or coincident but in 4 −fold 2 −valued Logic where P and Q can be both distinct and coincident.

4.4.2.2 Zeon's Arrow Paradox

A statement that leads to a contradictory conclusion from a sound argument that consists of premises and axioms is defined as a paradox [6]. If a paradox is derived in a system of knowledge, we have to reject the axioms that affect to the contradiction or to change the logic. Zeon's arrow paradox can be stated as follows. "If everything when it occupies an equal space is at rest, and if that which is in locomotion is always occupying such a space at any moment, the flying arrow is therefore motionless" [7]. "In this paradox, Zeno states that for motion to occur, an object must change the position which it occupies. He gives an example of an arrow in flight. He states that in any one (duration-less) instant of time, the arrow is neither moving to where it is, nor to where it is not. It cannot move to where it is not, because no time elapses for it to move there; it cannot move to where it is, because it is already there. In other words, at every instant of time there is no motion occurring. If everything is motionless at every instant, and time is entirely composed of instants, then motion is impossible. This paradox starts by dividing time-intervals and not into segments, but into points" [7]. Empirically, the arrow moves, but according to Aristotelian logic the conclusion is that arrow cannot move. That means there are no solutions for Zeno's paradoxes within the Aristotelian logic. Therefore, the conclusion forces us to change axioms or logic or primitive terms. In this case, we change the logic. Instead of Aristotelian logic, we apply $4-$fold $2-$valued logic. When the arrow moves, it occupies some space that is not constant in the time, the occupied space changes from moment to moment. On other hand, when it is at rest, it occupies a constant space in the time. While the arrow moves, it occupies some constant space as well as it does not occupy the same constant space at the same time. Suppose that the statement S is "the arrow occupies a space". This implies that S is true and $\neg S$ is true at the same time.

5. Conclusions

In the definition of many valued logics, the results of Aristotelian logic do not contradict. That means if a given statement is true, then the negated statement of the given statement is false and vice-versa. Every logic including classical two valued, many valued, fuzzy logic and Catuskoti can be expressed in the form of $n-$Fold $m-$Valued logic, where n, m integers that are greater than are or equal 2. We have identified that in the literature, logics other than classical two valued logic have been discussed. These logics are many valued logics, infinite valued logics, fuzzy logic and Catuskoti logic ($4-$fold $2-$valued logic). We have defined a new logic called $n-$fold $m-$valued logic such that all the logics are abstracted to the form of $n-$fold $m-$valued logic, where n and m are natural numbers that are greater than one. We have presented $4-$fold $2-$valued logic. The logicians have not gone beyond presenting truth tables. We propose to study the presentation of truth tables in detail for $n-$fold $m-$valued logic in the future.

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